

« Autour de l'oscillateur harmonique »

« About harmonic oscillator »

Cours de méthodologie – Ecole d'été (ITC Phnom Penh)

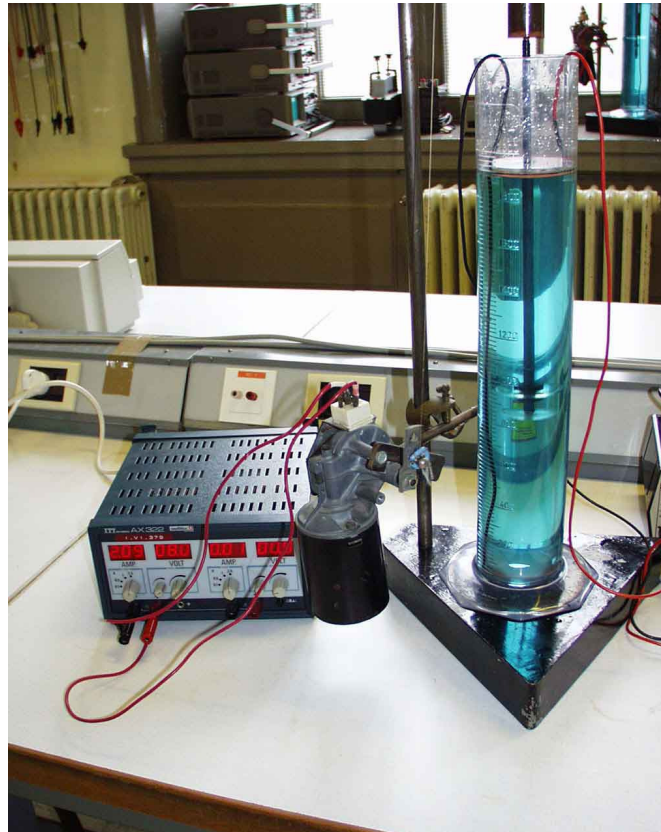
Olivier GRANIER

(Du lundi 20 au mercredi 29 août 2012)

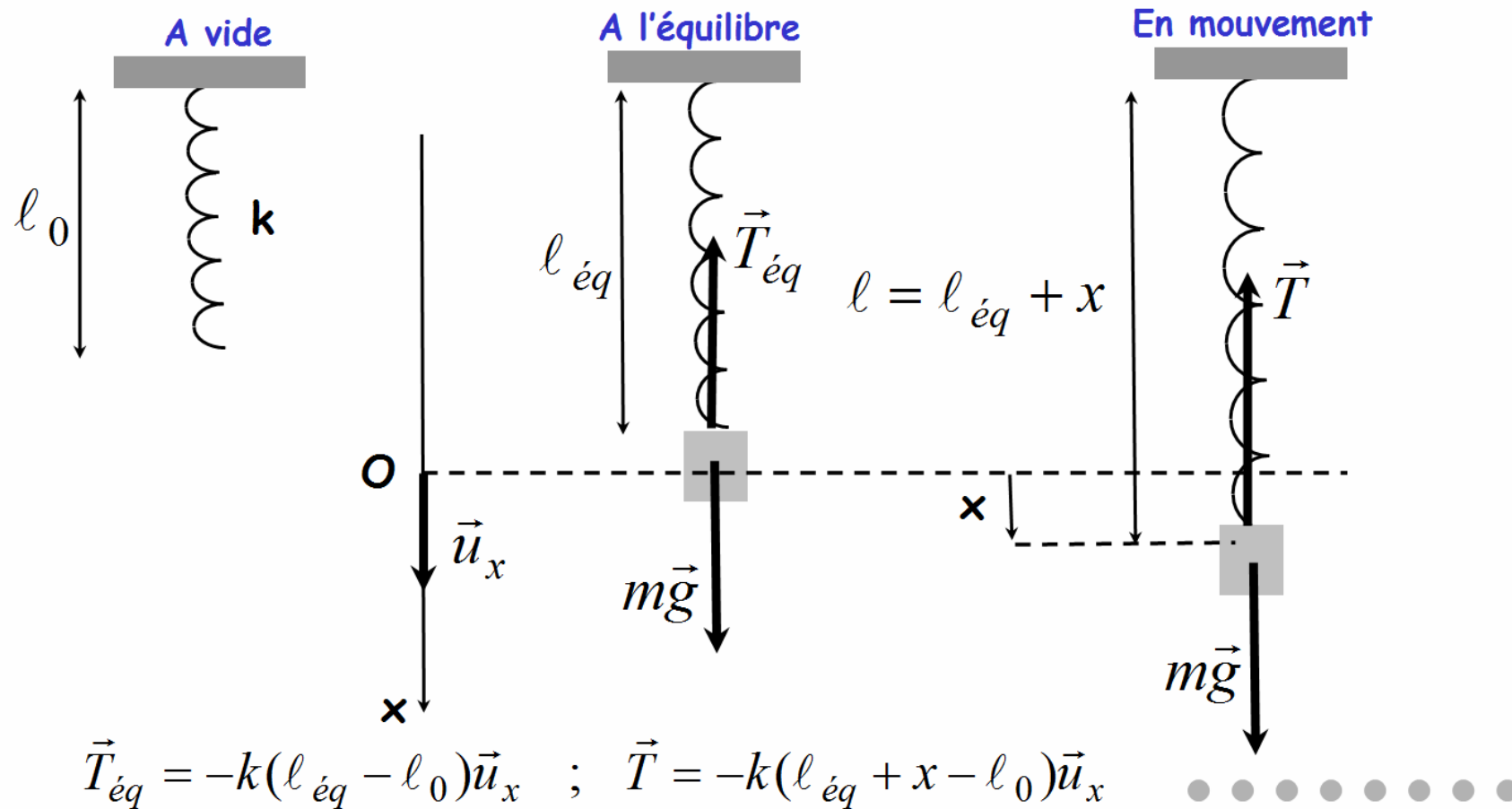
1 - Le système masse – ressort : (Spring – Mass system)

Un TP sur l'oscillateur harmonique mécanique

Harmonic oscillator in mechanics



En régime libre (Simple damped harmonic oscillator)



In real oscillators, friction, or damping, slows the system's motion.

In many vibrating systems the frictional force f_d can be modeled as being proportional to the object's velocity v :

$$\vec{f}_d = -h\vec{v}$$

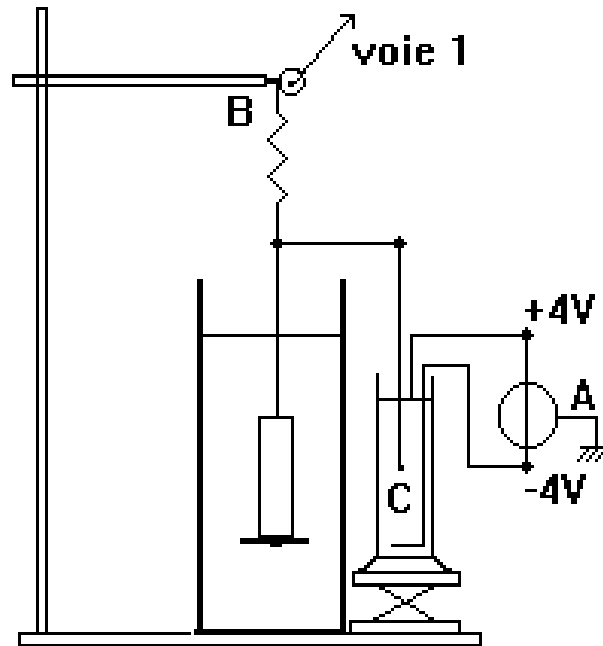
h : viscous damping coefficient.

Newton's second law :

$$m \frac{d^2 x(t)}{dt^2} \vec{u}_x = -k(l_{eq} + x - \ell_0) \vec{u}_x - h\vec{v} + m\vec{g}$$

$$m\ddot{x} + h\dot{x} + kx = 0$$

Vidéo : « Relaxation d'un ressort »



$$\ddot{x} + 2\sigma\omega_0\dot{x} + \omega_0^2 x = 0$$

$$\ddot{x} + \frac{\omega_0}{Q}\dot{x} + \omega_0^2 x = 0$$

$$\omega_0 = \sqrt{\frac{k}{m}}$$

;

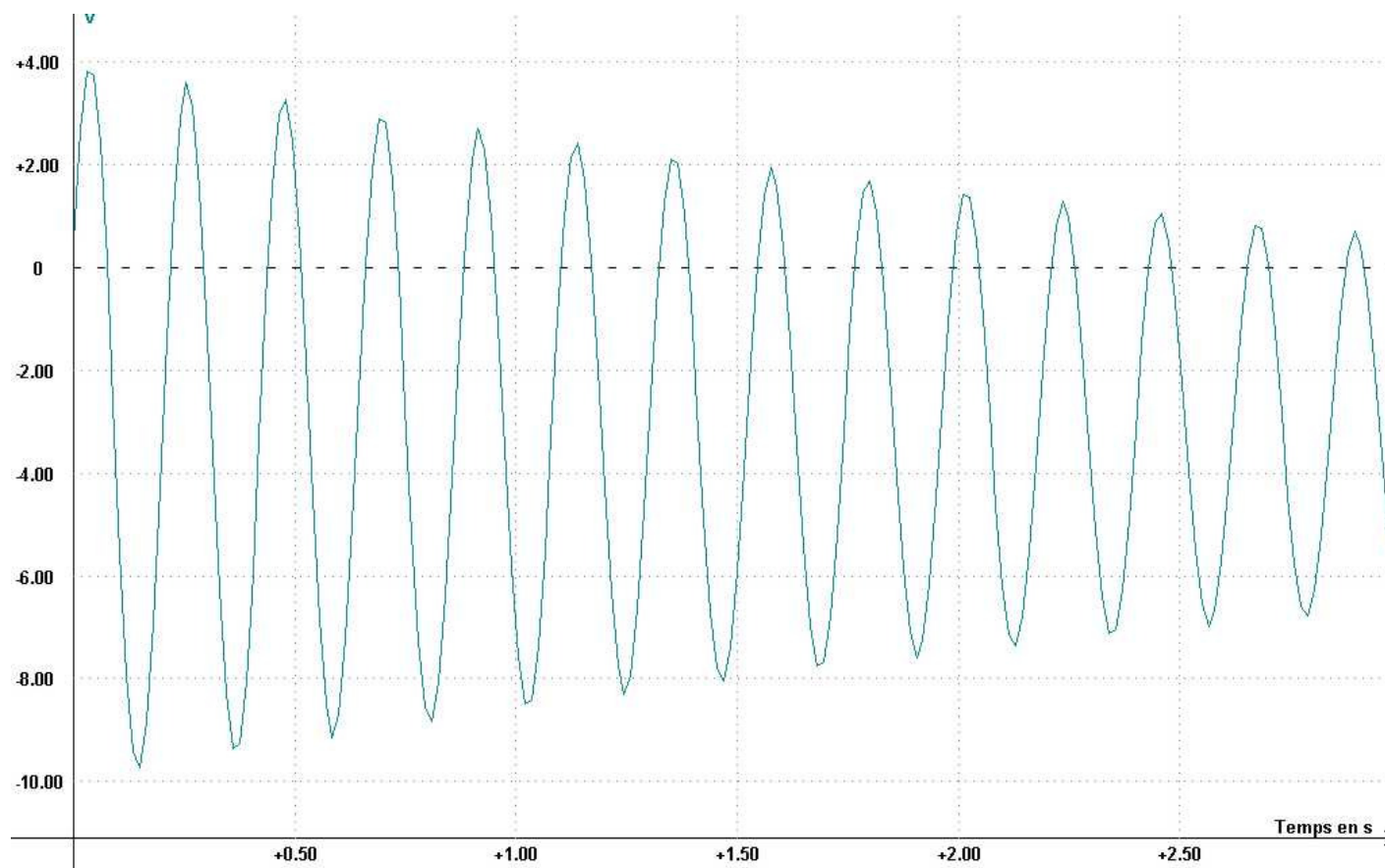
$$2\sigma\omega_0 = \frac{h}{m}$$

;

$$\sigma = \frac{1}{2} \frac{h}{\sqrt{km}}$$

$$x(t) = Ae^{-\sigma\omega_0 t} \sin[\sqrt{1-\sigma^2}\omega_0 t + \varphi] = Ae^{-\sigma\omega_0 t} \sin[\Omega t + \varphi]$$

$$(\Delta = 4\omega_0^2(\sigma^2 - 1) < 0)$$



En régime sinusoïdal forcé sinusoïdal

Driven harmonic oscillator : sine-shaped driving force





Tacoma Narrows Bridge Collapse (7/11/1940)
Lecture 27 MIT : resonance, destructive resonance, 13 : 30)

How to solve a differential equation with Regressi software ?

We want to solve, by a numerical way, the differential equation :

$$\ddot{x} + \frac{\omega_0}{Q} \dot{x} + \omega_0^2 x = \omega_0^2 X_0 \cos(\omega t)$$

With :

$$\omega_0 = 1 \text{ rad.s}^{-1} ; Q \text{ between } 0,1 \text{ and } 10 ; \omega \in [0.05 \text{ rad.s}^{-1}; 5 \text{ rad.s}^{-1}]$$

 Ajouter  Syntaxe  MàJ  Imprimer  Copier  Radian

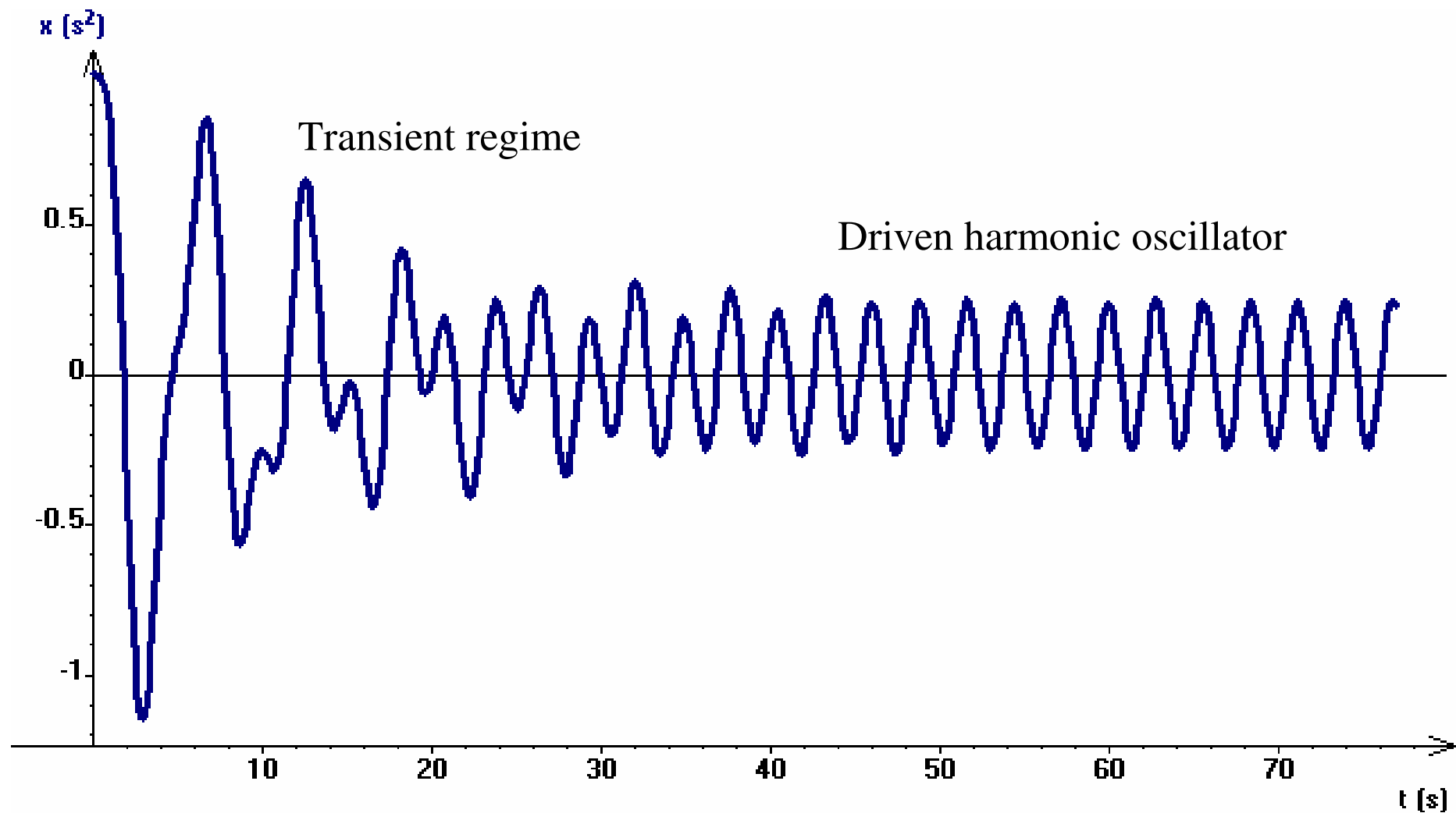
Variable de contrôle

Nom	t	Mini	0	Npoints	2048	☐ Remplissage exponentiel
Unité	s	Maxi	77	Δt	0.0376	Echantillonnage : f=26.5974 Hz

Constantes

Statistique

X0=1 -> X0=1
x''=-(w0/Q)*x'-(w0*w0)*x+(w0*w0)*X0*cos(w*t)

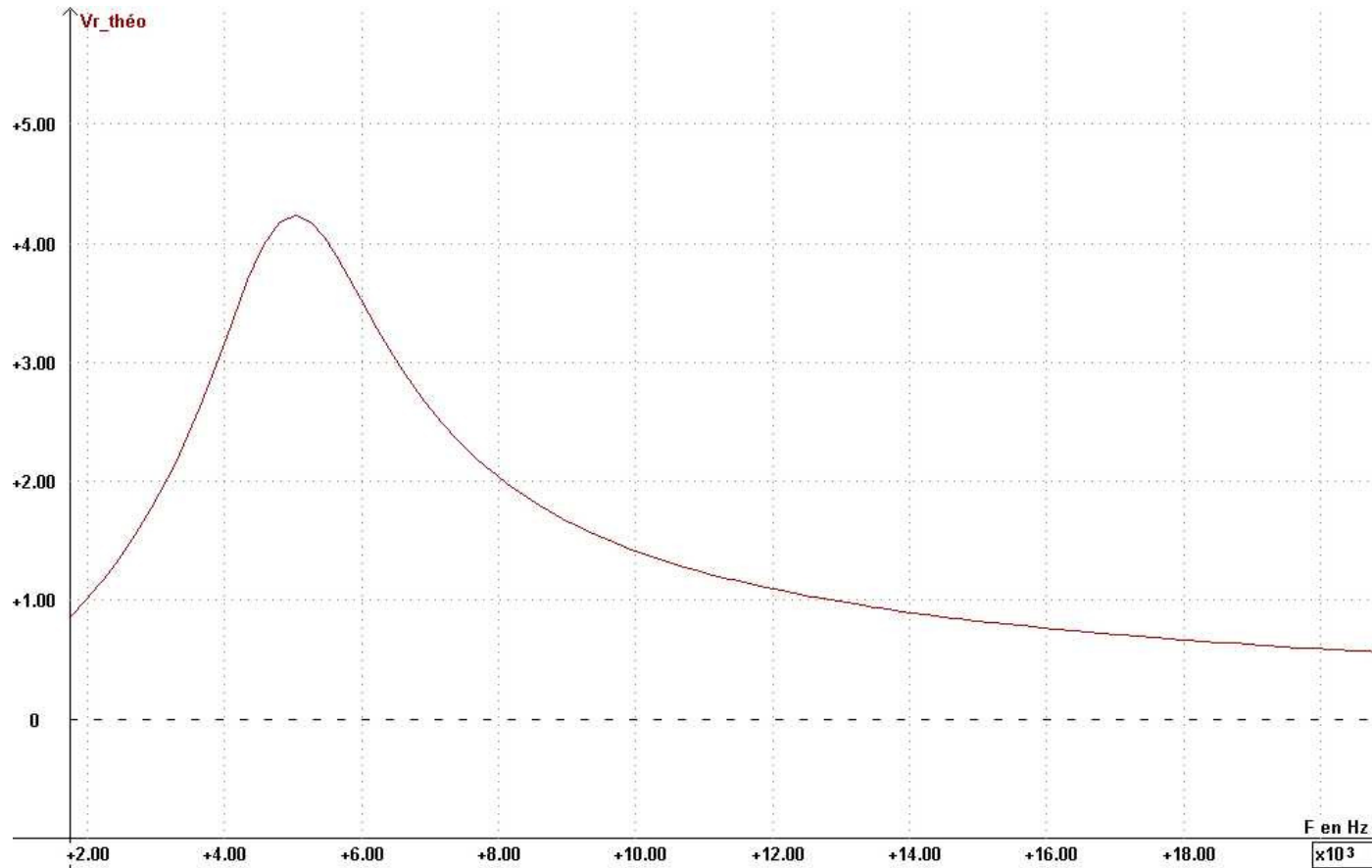


File (with Regressi software) : [resonance-eq-diff-resolue](#)

Experimental resonance's plot

Maximum of amplitude (resonance) is obtained at the angular frequency :

$$\omega_r = \omega_0 \sqrt{1 - \frac{1}{2Q^2}} = \omega_0 \sqrt{1 - 2\sigma^2} \quad \left(\text{Assuming : } Q > \frac{1}{\sqrt{2}} \right)$$



Amplitude du mouvement du régime forcé : (amplitude of damped sine-shaped oscillations)

$$X_m(\omega) = \frac{X_0}{\sqrt{\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2 + 4\sigma^2 \frac{\omega^2}{\omega_0^2}}}$$

Cette amplitude est maximale pour : (there is resonance when)

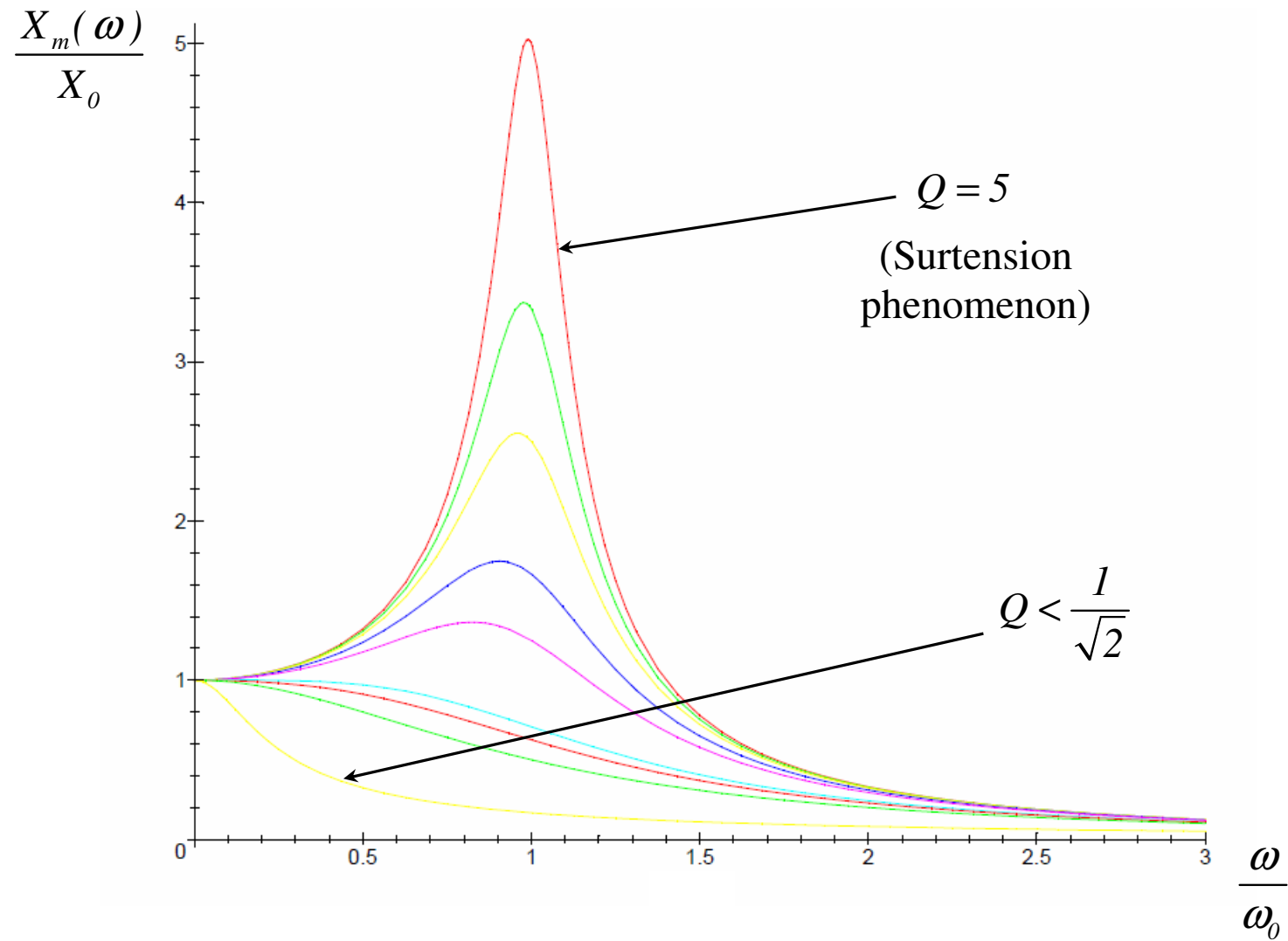
$$\omega_r = \omega_0 \sqrt{1 - \frac{1}{2Q^2}} = \omega_0 \sqrt{1 - 2\sigma^2}$$

La vitesse est : (the velocity is)

$$\underline{v} = V_m e^{j(\omega t + \psi)} = \underline{\dot{x}} = j\omega X_m e^{j(\omega t + \varphi)}$$

$$V_m(\omega) = \frac{\omega X_0}{\sqrt{\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2 + 4\sigma^2 \frac{\omega^2}{\omega_0^2}}} = \frac{\omega_0 X_0}{\sqrt{\left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0}\right)^2 + 4\sigma^2}}$$

Courbe de réponse en amplitude X_m (Plot $X_m(\omega)$)



Vidéo : « [Résonance de pendules](#) »

Animations JJR

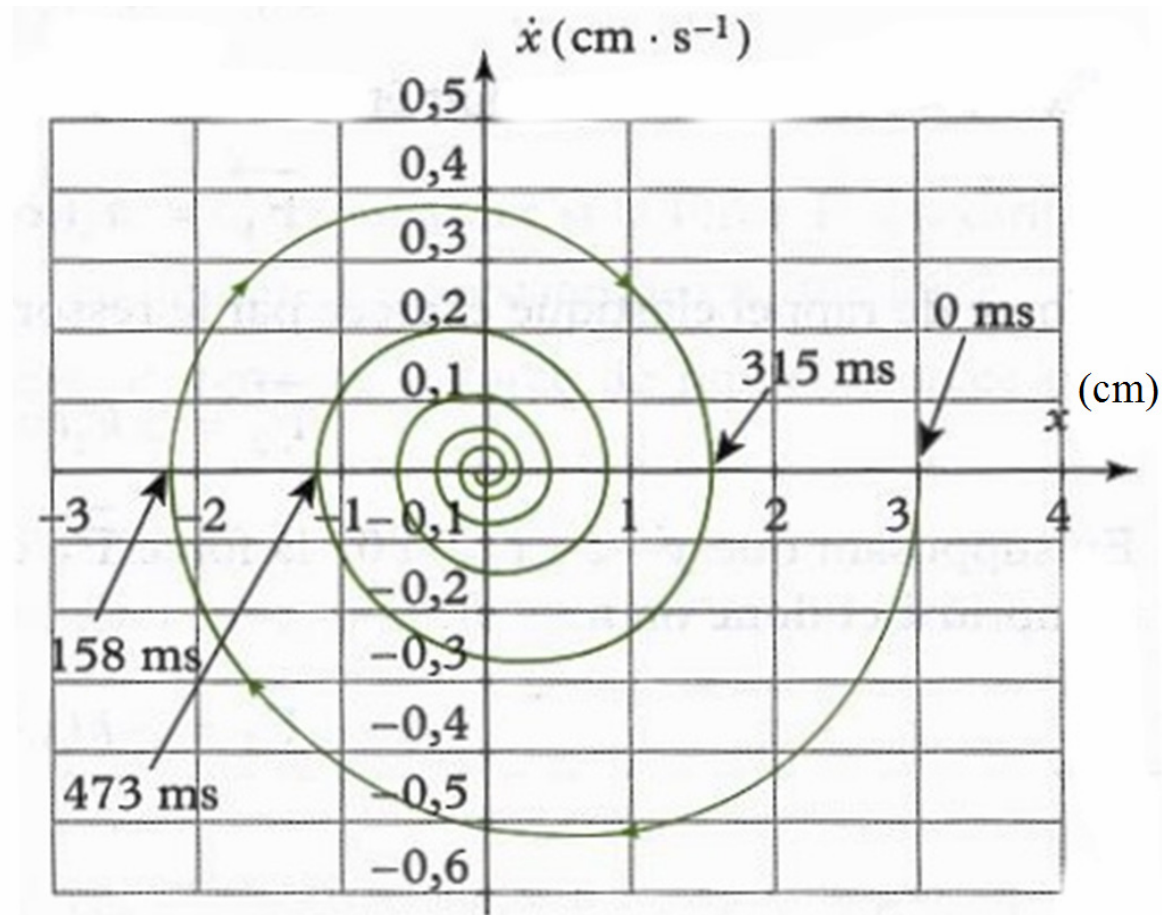
Oscillateur à ressort (portrait de phase) : (Spring – Mass system and phase portrait)

[JJR/Mécanique/Oscillateurs/Oscillateur harmonique](#)

Interprétation du facteur de qualité : $Q \approx$ nombre d'oscillations visibles

Interpretation of the Q factor : $Q \approx$ number of oscillations seen on the plot $x(t)$

[JJR/Mécanique/Oscillateurs/Oscillateur harmonique excité](#)



($m = 500$ g)

Programme Regressi (Lecture d'un portrait de phase)

Calculate : the undamped angular frequency ω_0 , the Q factor, the spring constant k and the viscous damping coefficient h .

Pseudo – period of the underdamped oscillator regime : $T = 315 \text{ ms}$

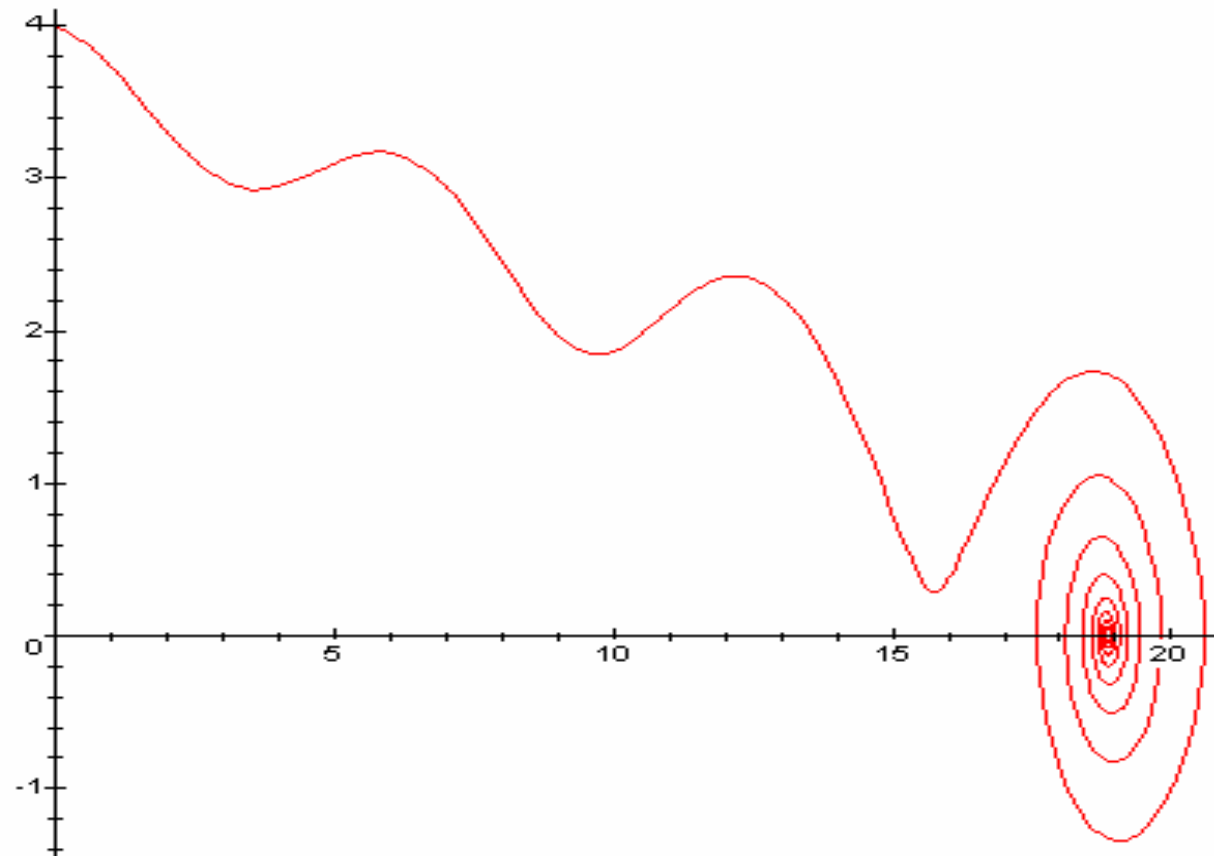
$$\omega_0 = \sqrt{\frac{k}{m}} \quad ; \quad T_0 = \frac{2\pi}{\omega_0} \quad ; \quad T = T_0 \sqrt{1 - \frac{1}{4Q^2}} \quad ; \quad \frac{\omega_0}{Q} = \frac{h}{m}$$

We see 5 oscillations $\Rightarrow Q \approx 5$

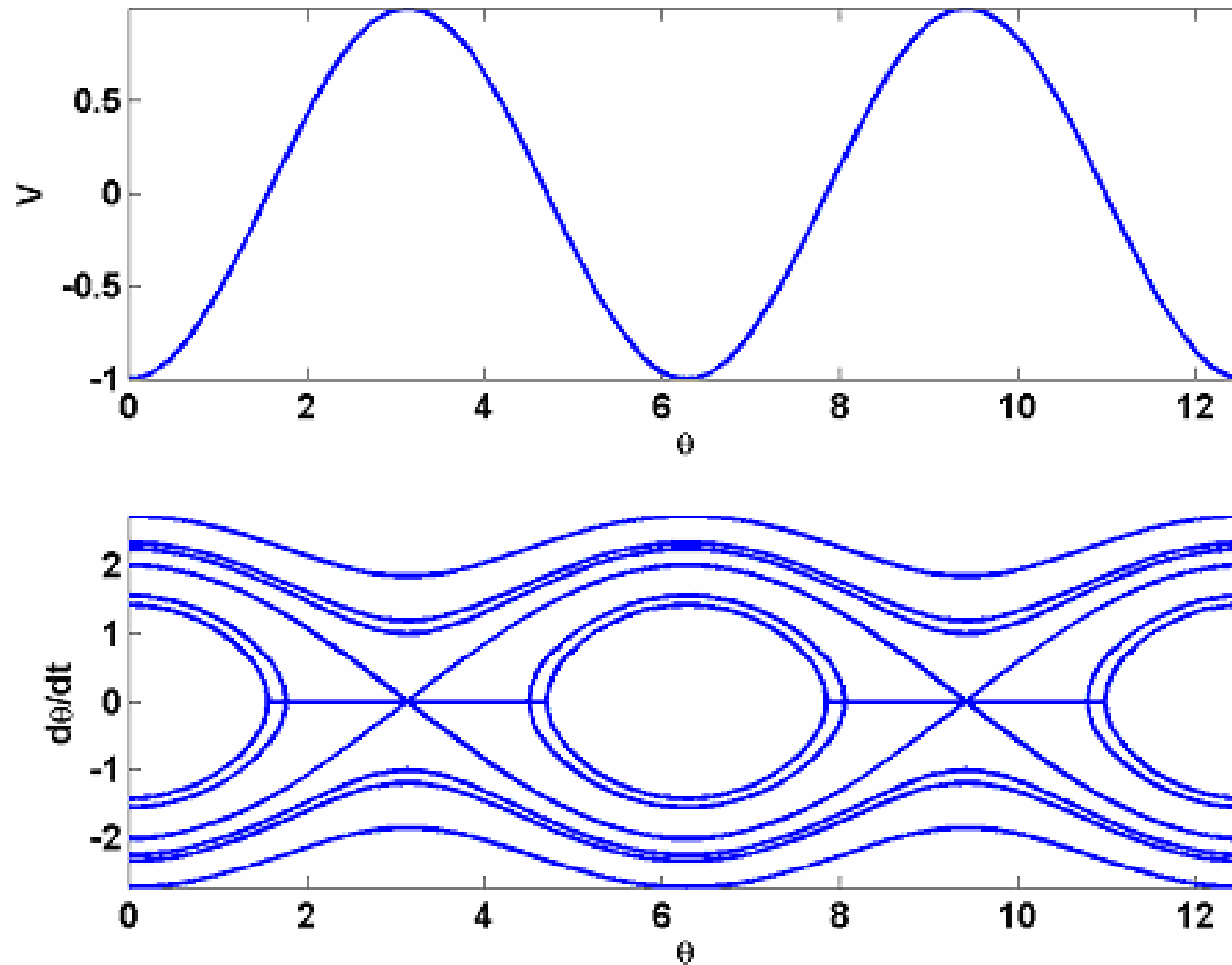
$$T = T_0 \sqrt{1 - \frac{1}{4Q^2}} \approx T_0 \quad \Rightarrow \quad \omega_0 = \frac{2\pi}{T} = 19,9 \text{ rad.s}^{-1} \text{ and } k = \frac{4\pi^2 m}{T^2} = 198,7 \text{ N.m}^{-1}$$

$$\exp\left(-\frac{\omega_0}{2Q}T\right) = \frac{1,6}{3} \quad \Rightarrow \quad Q = \frac{1}{\ln(3/1,6)} \frac{\omega_0 T}{2} = \frac{\pi}{\ln(3/1,6)} = 5$$

$$h = \frac{m\omega_0}{Q} = 2 \text{ N.m}^{-1}.\text{s}$$



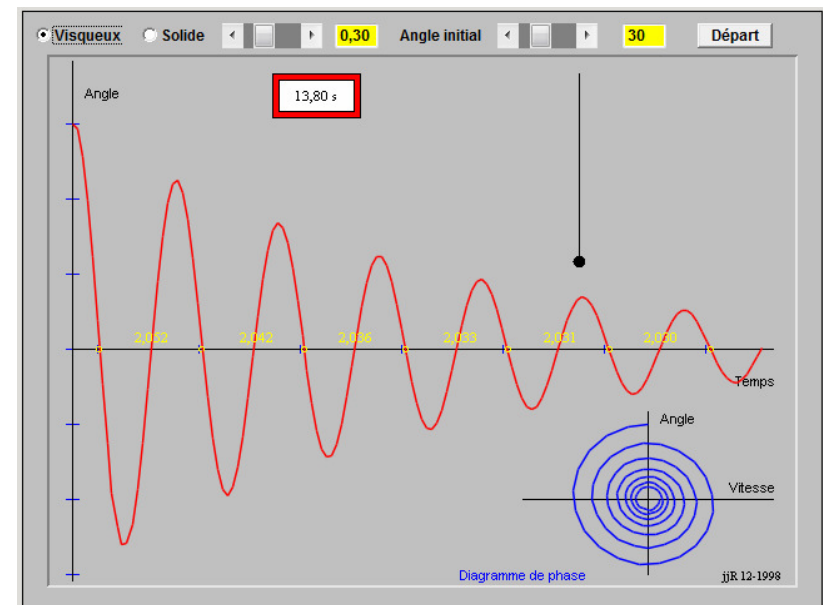
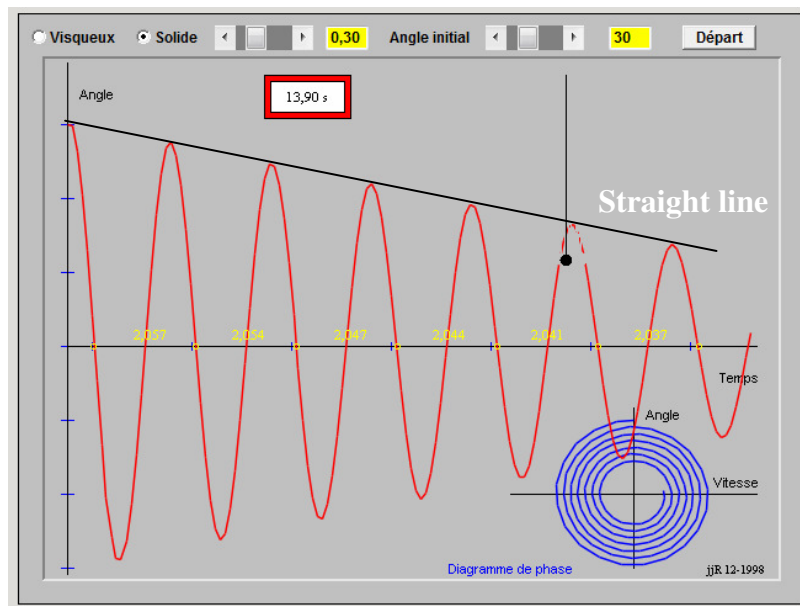
Regressi : « Phase portrait of a simple damped pendulum »



*Potential energy and phase portrait of a simple pendulum (no friction).
The x-axis, being angular, wraps onto itself after every 2π radians.*

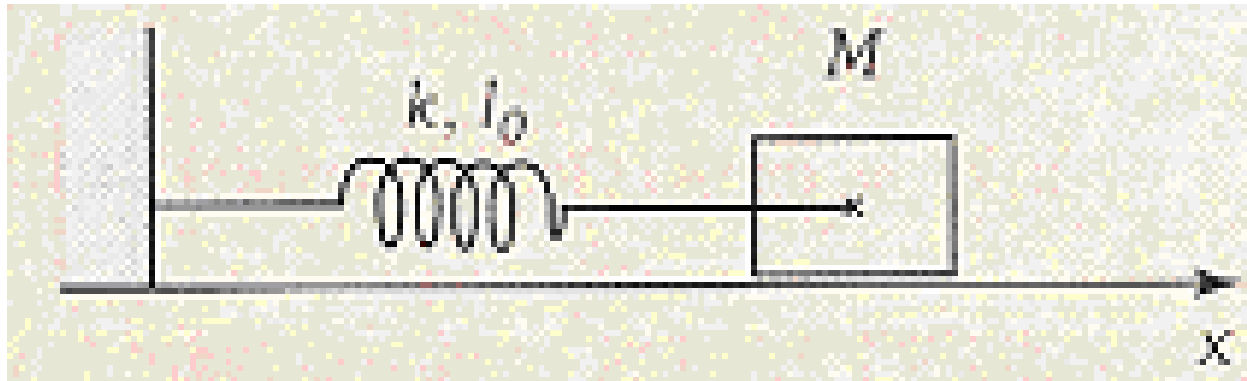
Comparaison frottements solide/fluide : (Solid or Coulomb friction vs viscous damping)

JJR/Mécanique/Oscillateurs/Pendule amorti



A cube (mass M) can slide on an horizontal area with a solid-friction coefficient f .

At $t = 0$: the spring's length is $\ell_0 + A$ ($A > 0$) and the velocity equals zero.



- * What is the condition for the cube to be set to motion ?
- * If the cube is set to motion, describe the motion and give $x(t)$ until the cube stops.
- * What is the new condition for the cube to be set to motion again ?
- * Supposing there are plenty of oscillations, what is the $x(t)$ plot's envelope ? Compare with the viscous damping.

The mass is moving if :

$$A > \frac{fmg}{k}$$

$$0 < t < \frac{T}{2} = \frac{\pi}{\Omega} \left(\Omega = \sqrt{\frac{k}{M}} \right) : \quad \boxed{x(t) = \left(A - \frac{fmg}{k} \right) \cos(\Omega t) + \frac{fmg}{k}}$$

For $t = T/2$:

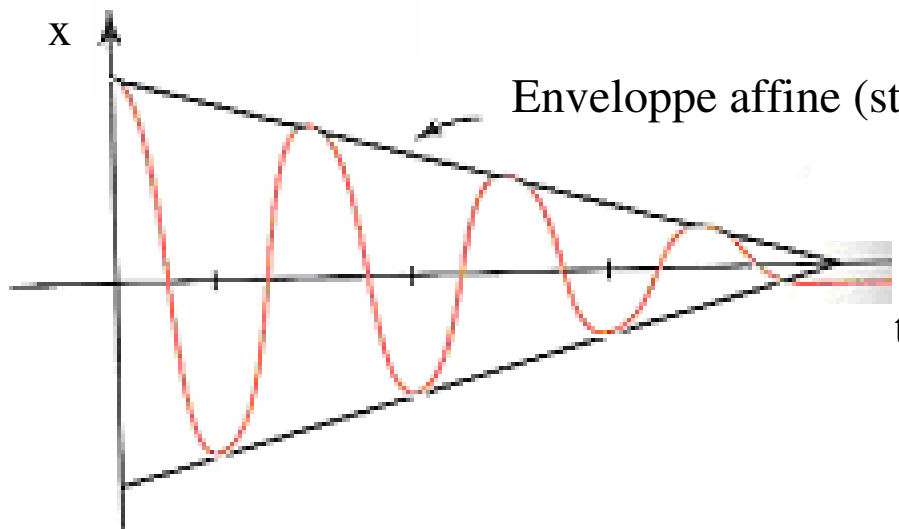
$$x\left(\frac{T}{2}\right) = -A + 2 \frac{fmg}{k}$$

If $A > 3 \frac{fmg}{k}$:

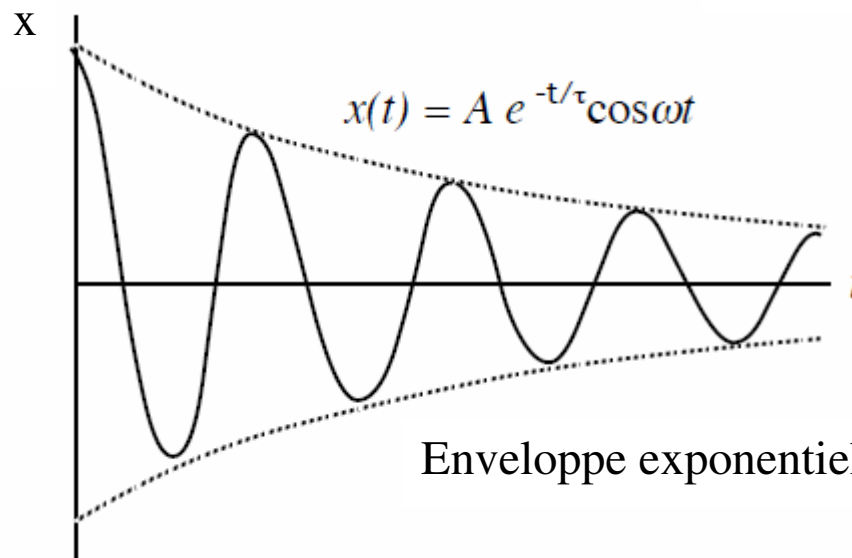
$$\frac{T}{2} < t < T : \quad \boxed{x(t) = \left(A - 3 \frac{fmg}{k} \right) \cos(\Omega t) - \frac{fmg}{k}}$$

For $t = T$:

$$x(T) = A - 4 \frac{fmg}{k}$$



Solid or Coulomb friction



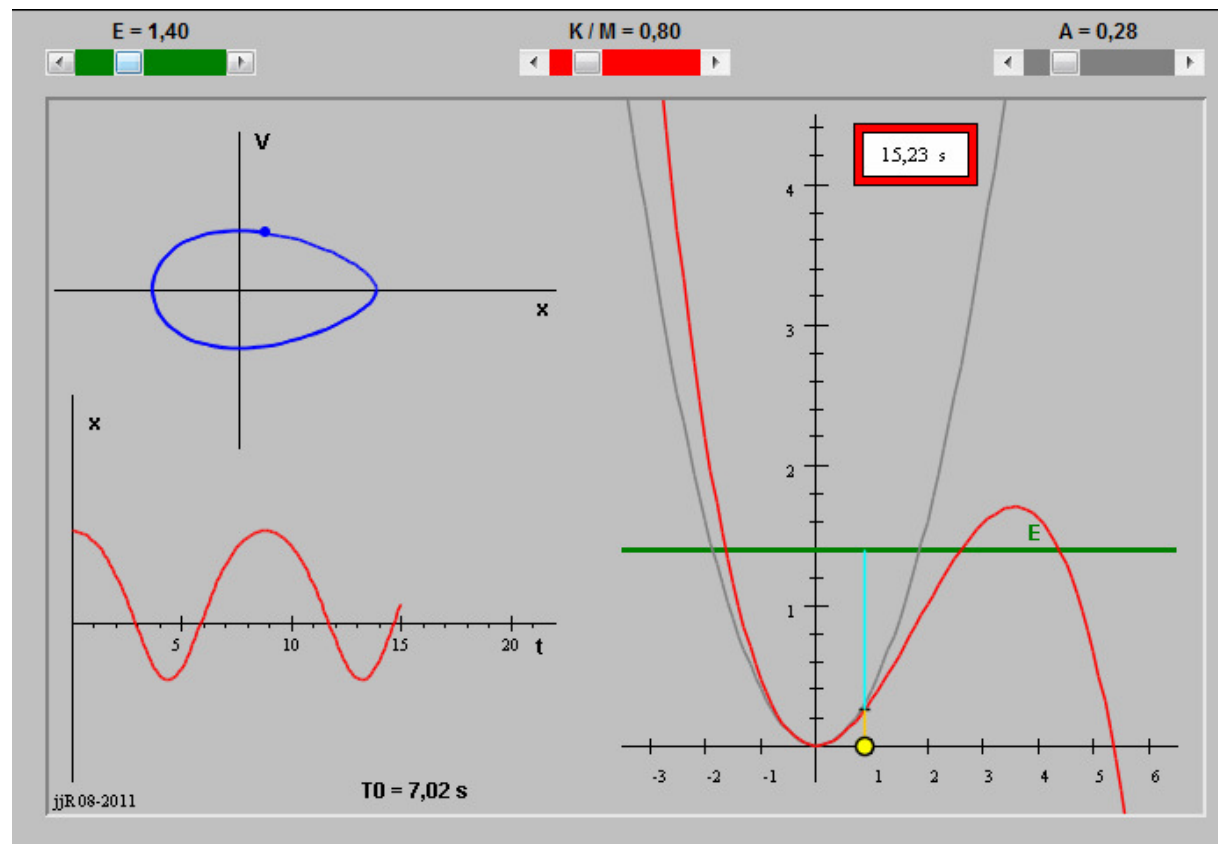
Viscous damping

Enveloppe exponentielle (exponentiel envelope)

2 – Intérêt du modèle de l'oscillateur harmonique : (Application to a conservative force)

Oscillateur à ressort (phase portrait) :

[JJR/Mécanique/Oscillateurs/oscilanh](#)



The problem of the simple harmonic oscillator occurs frequently in physics.

A mass at equilibrium under the influence of any conservative force, in the limit of small motions, behaves as a simple harmonic oscillator.

Exemple :

Potentiel energy can be written as :

$$\boxed{E_p(x) = \frac{1}{2} Kx^2 - \frac{1}{3} AKx^3} \quad \text{or} \quad \boxed{E_p(x) = \frac{A}{x^2} - \frac{B}{x}} \quad (\text{Molecular potential})$$

The harmonic oscillator model is very important in physics, because any mass subject to a force in stable equilibrium acts as a harmonic oscillator for small vibrations.

Harmonic oscillators can be found in nature (“ why the sky is blue ? ”) and in objects of everyday life, such as clocks and radio circuits.

They are the source of virtually all sine – shaped (sinusoidal) vibrations and waves

3 – Un modèle électrique : le circuit série RLC (Resistance – Inductor – Capacitor) :

a) Régime transitoire : (Transient response)

$$u_L + u_R + u_C = L \frac{di}{dt} + Ri + \frac{1}{C} q = e(t)$$

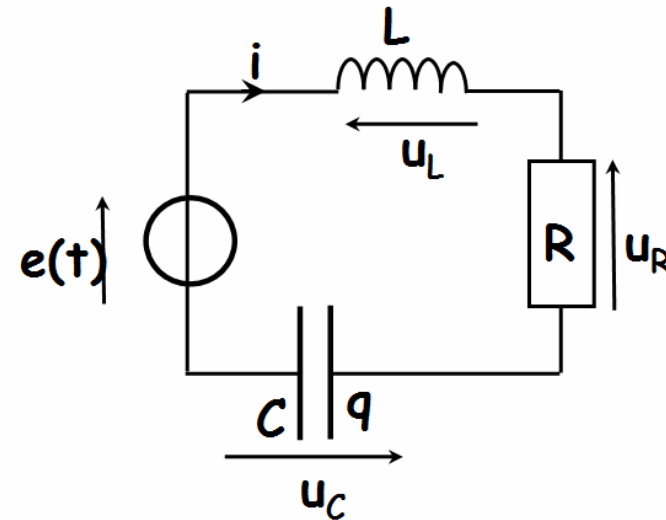
$$i = \dot{q} \quad ; \quad u_C = \frac{q}{C} \quad \text{donc} \quad i = C \frac{du_C}{dt}$$

$$LC \frac{d^2 u_C}{dt^2} + RC \frac{du_C}{dt} + u_C = e(t)$$

$$\frac{d^2 u_C}{dt^2} + \frac{R}{L} \frac{du_C}{dt} + \frac{1}{LC} u_C = \frac{1}{LC} e(t)$$

$$\frac{d^2 u_C}{dt^2} + 2\sigma\omega_0 \frac{du_C}{dt} + \omega_0^2 u_C = \omega_0^2 e(t)$$

$$\text{avec} \quad \omega_0^2 = \frac{1}{LC} \quad \text{et} \quad 2\sigma\omega_0 = \frac{R}{L}$$



$$\frac{d^2 u_c}{dt^2} + 2\sigma\omega_0 \frac{du_c}{dt} + \omega_0^2 u_c = \frac{d^2 u_c}{dt^2} + \frac{\omega_0}{Q} \frac{du_c}{dt} + \omega_0^2 u_c = \omega_0^2 e(t)$$

characteristic equation of the form :

$$(\Delta = 4\omega_0^2(\sigma^2 - 1)) \quad r^2 + 2\sigma\omega_0 r + \omega_0^2 = 0$$

$\Delta < 0$ soit $\sigma < 1$: régime pseudo – périodique (underdamped)
 $(r_1, r_2 \in \mathbb{C})$

The system oscillates (with a slightly different frequency than the undamped case) with the amplitude gradually decreasing to zero.

The angular frequency of the underdamped harmonic oscillator is given by :

$$\omega = \sqrt{1 - \sigma^2} \omega_0 = \sqrt{1 - \frac{1}{4Q^2}} \omega_0$$

Q : Q factor (quality factor) :

$$Q = \frac{1}{2\sigma} = 2\pi \frac{\text{Energy stored}}{\text{Energy lost every cycle}}$$

$\Delta = 0$ soit $\sigma = 1$: régime apériodique critique (critically damped)
(single solution, $r = -\omega_0$)

The system returns to equilibrium as quickly as possible without oscillating.

$\Delta > 0$ soit $\sigma > 1$: régime apériodique (overdamped)
($r_1, r_2 \in \mathbb{R}$)

The system returns to equilibrium without oscillating.

Larger values of the damping ratio σ return to equilibrium in a slower way.

Réponse du circuit RLC à un échelon de tension : (step response)

Pour $t > 0$:

$$\frac{d^2 u_C}{dt^2} + 2\sigma\omega_0 \frac{du_C}{dt} + \omega_0^2 u_C = \omega_0^2 E$$

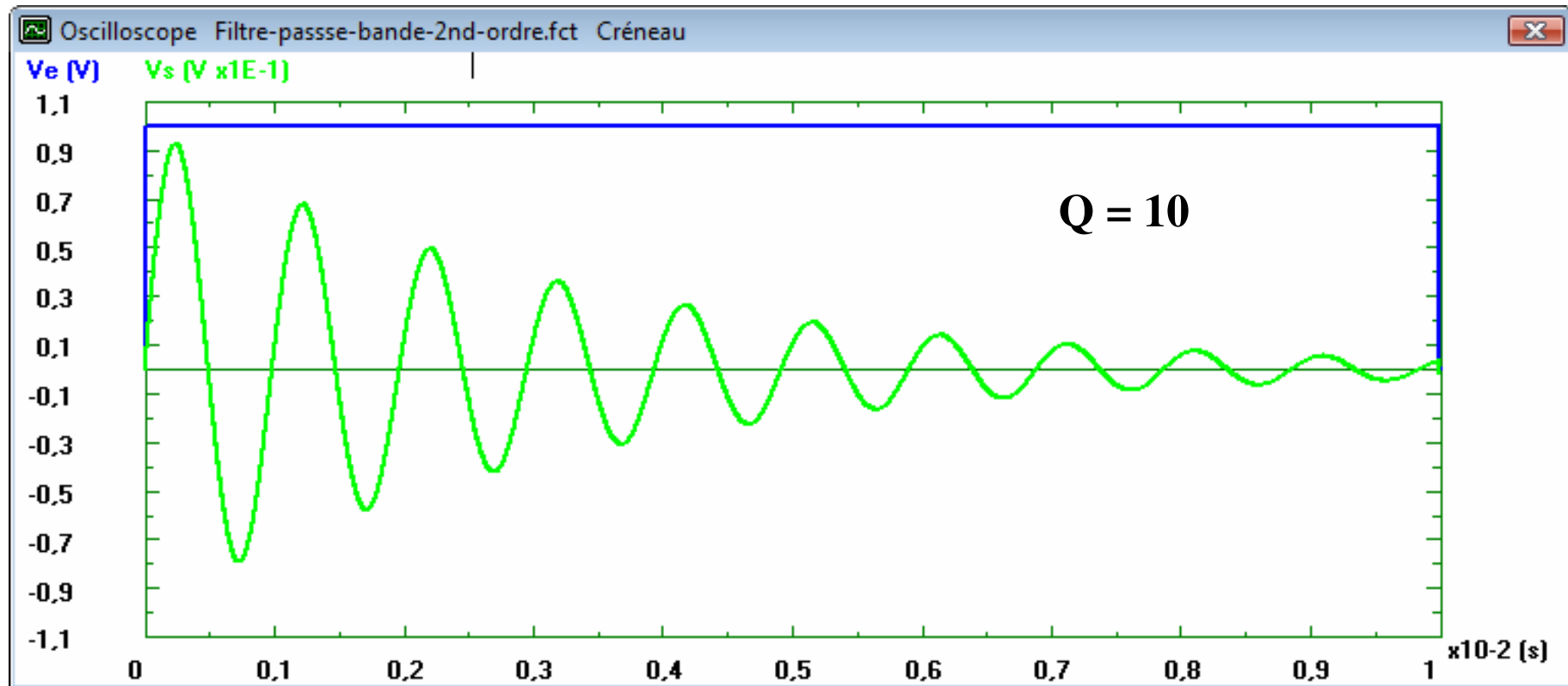
$$\sigma < 1 : u_C = e^{-\sigma\omega_0 t} \left[A \cos(\omega_0 \sqrt{1-\sigma^2} t) + B \sin(\omega_0 \sqrt{1-\sigma^2} t) \right] + E$$

$$\sigma > 1 : u_C = e^{-\sigma\omega_0 t} \left[A e^{(\omega_0 \sqrt{\sigma^2-1})t} + B e^{(-\omega_0 \sqrt{\sigma^2-1})t} \right] + E$$

$$\sigma = 1 : u_C = e^{-\omega_0 t} [A + Bt] + E$$

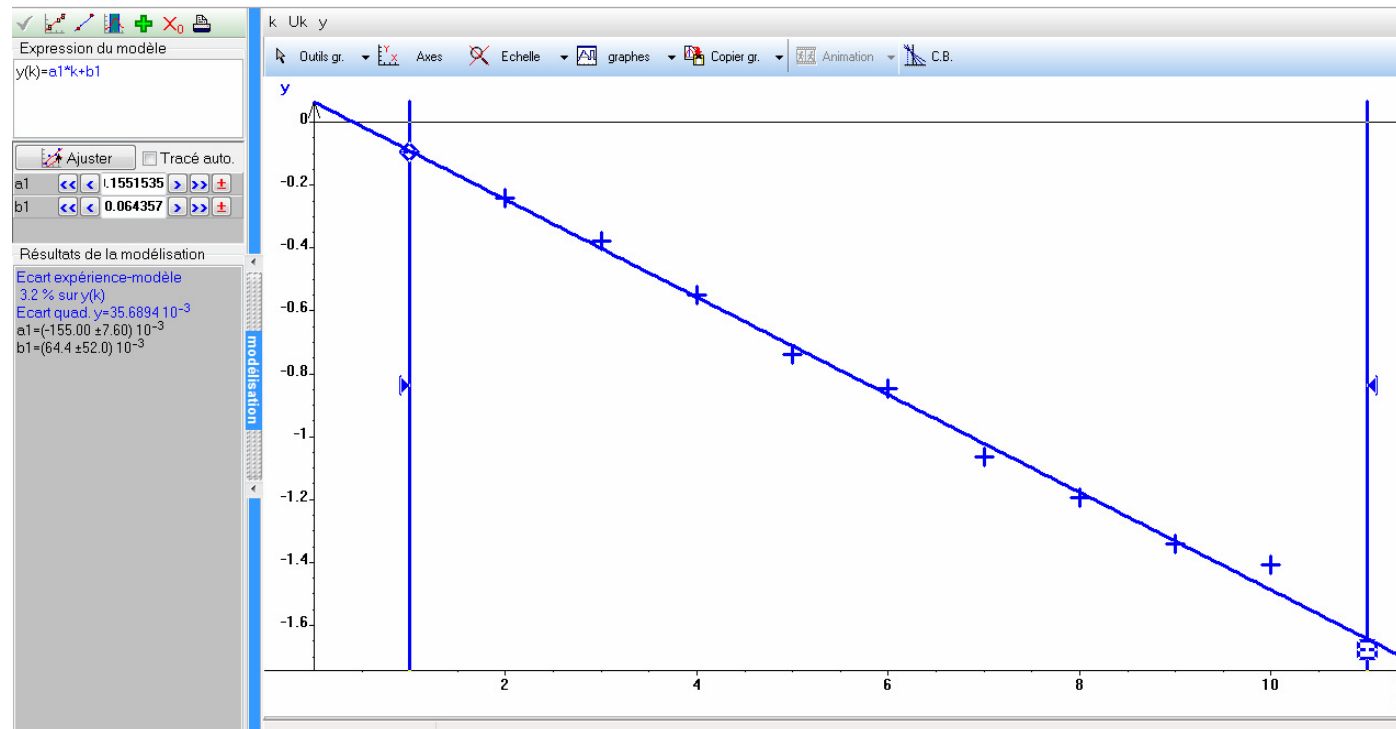
$$\tau = \frac{1}{\sigma\omega_0} : \text{relaxation time}$$

File you can make with the “AnHarm” software : [Filtre-passe-bande-2nd-ordre.fct](#)



(We can see about 10 pseudo-oscillations with $C = 10^{-7}$ F)

Décrément logarithmique : (logarithmic decrease)



Paramètres			Vari
Trier	Ajouter	Sup.	
i	k	Uk	V
0			
1	1.000000	0.9100000	
2	2.000000	0.7850000	
3	3.000000	0.6850000	
4	4.000000	0.5770000	
5	5.000000	0.4780000	
6	6.000000	0.4280000	
7	7.000000	0.3450000	
8	8.000000	0.3030000	
9	9.000000	0.2620000	
10	10.00000	0.2450000	
11	11.00000	0.1870000	
12			

File made with Regressi software

Theoretical study :

Les amortissements sont faibles (friction is weak) $\left(\omega_0 = \frac{1}{\sqrt{LC}} \text{ et } \frac{\omega_0}{Q} = \frac{R}{L} \right) :$

$$v_s(t) = V_m e^{-\frac{\omega_0}{2Q}t} \sin(\omega_0 t)$$

For the different moments :

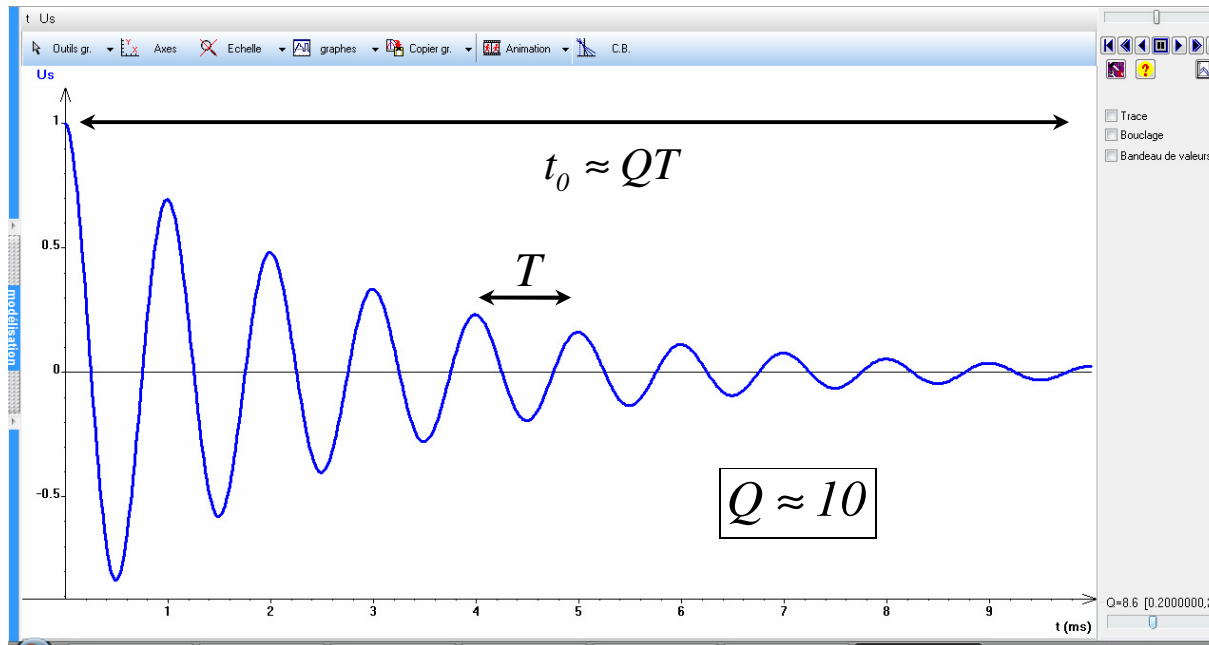
$$t_k = (2k+1) \frac{T_0}{4} \quad (T_0 = \frac{2\pi}{\omega_0}) \quad : \quad |v_s(t_k)| = V_m e^{-\frac{\omega_0}{2Q}t_k} = V_m e^{-\frac{\pi}{4Q}(2k+1)}$$

$$y_k = \ln |v_s(t_k)| = \ln(V_m) - \frac{\pi}{4Q}(2k+1)$$

With Regressi software :

$$\frac{\pi}{4Q} 2k = 0,155 \quad ; \quad Q \approx 10$$

Interprétation du facteur de qualité : (Interpretation of the Q factor)



Regressi software

$$v_s(t) = V_m e^{-\frac{\omega_0}{2Q}t} \cos(\omega_0 t) = V_m e^{-\frac{t}{\tau}} \cos(\omega_0 t)$$

At t_0 :

$$e^{-\frac{t_0}{\tau}} \approx e^{-3} = 0,05 \approx 0 \quad \text{hence} \quad t_0 = 3\tau = 3 \frac{2Q}{\omega_0}$$

Underdamped regime :

$$T = T_0 \sqrt{1 - \frac{1}{4Q^2}} \approx T_0 = \frac{2\pi}{\omega_0} \Rightarrow t_0 = \frac{3}{\pi} QT \approx QT$$

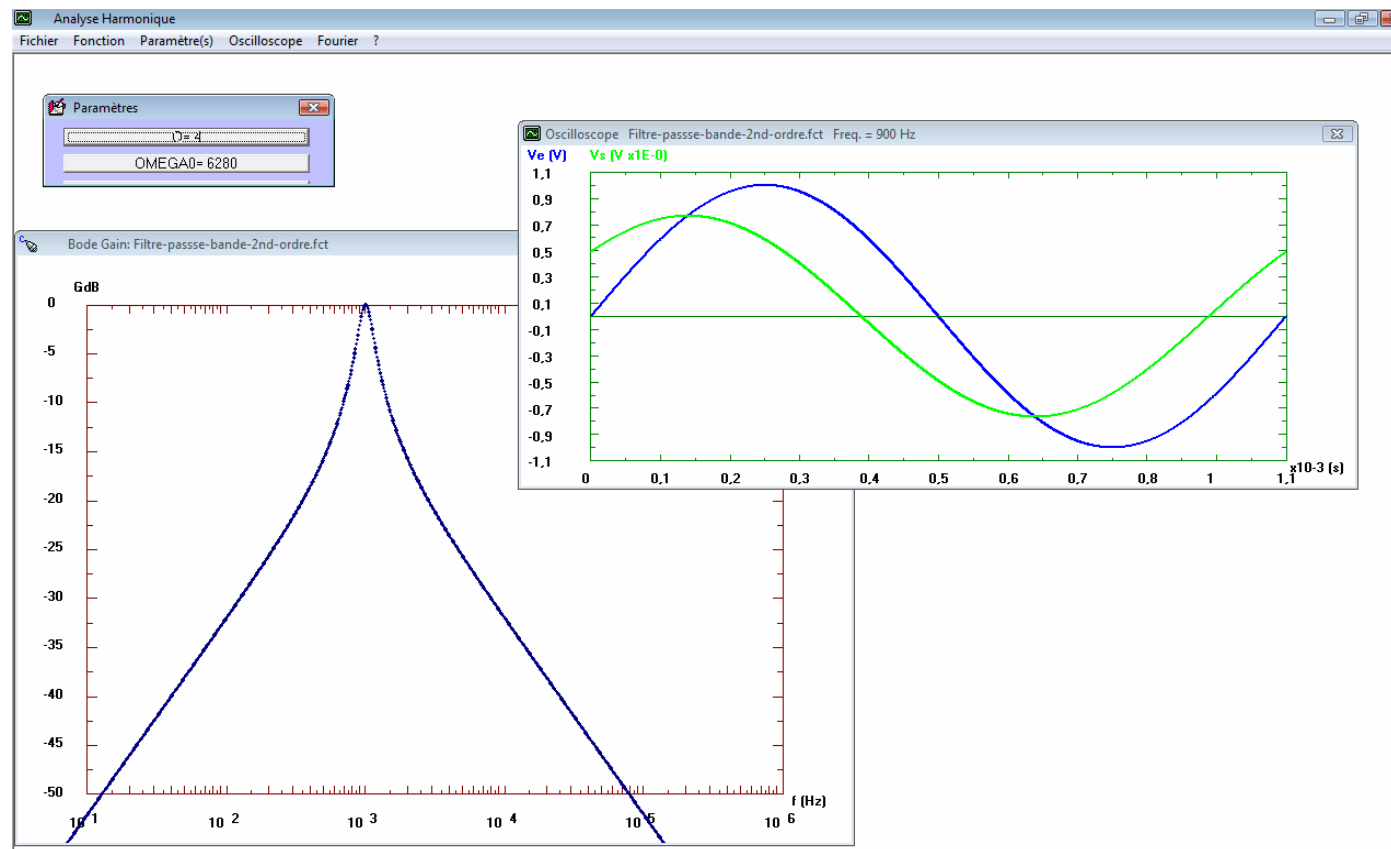
- Mesures de R et de L : (Experimental results for R and L, with $C = 10^{-7} \text{ F}$)

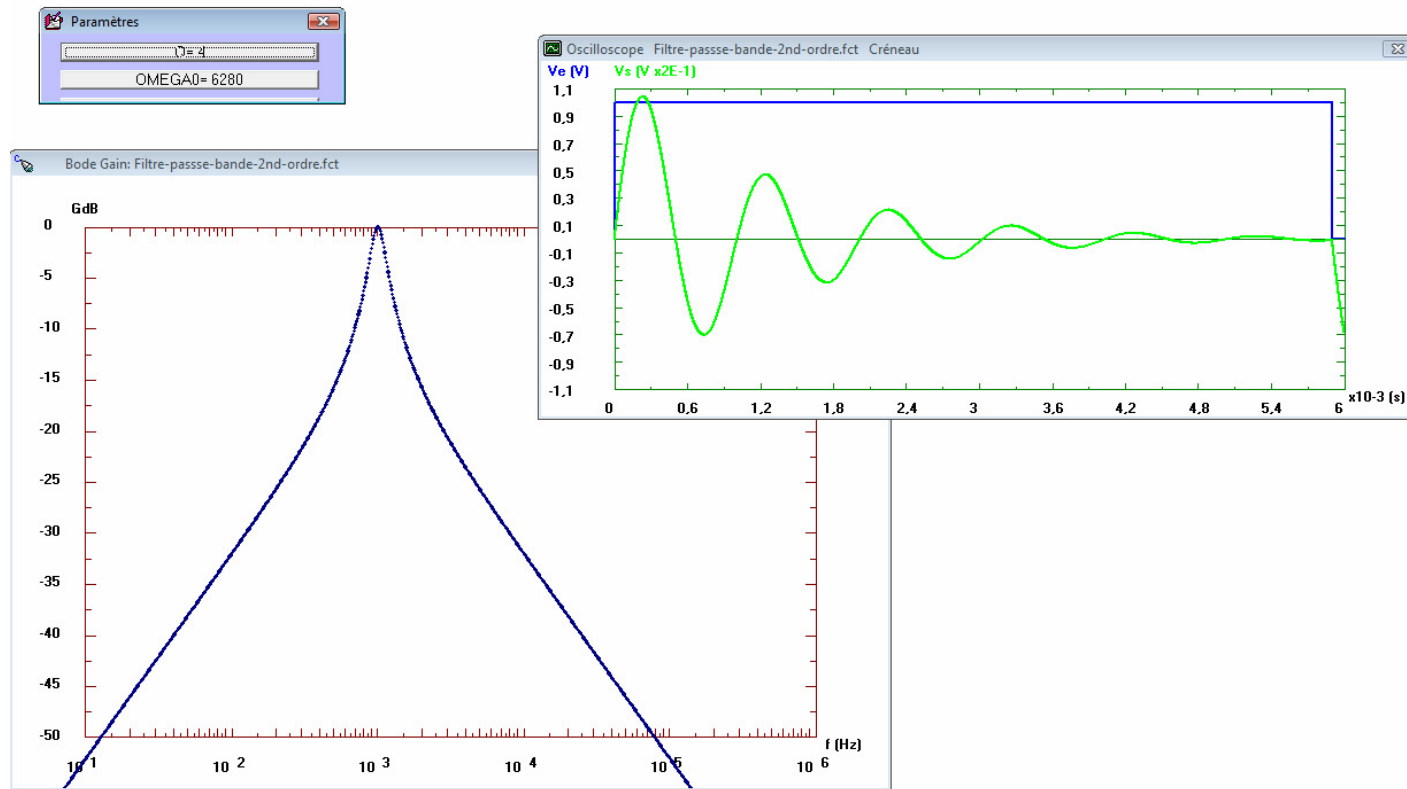
$$T_0 \approx \frac{10^{-2}}{10} \text{ s} = 1 \text{ ms} \quad ; \quad L = \frac{1}{C\omega_0^2} = \frac{T_0^2}{4\pi^2 C} \approx 0,25 \text{ H} \quad ; \quad \frac{\omega_0}{Q} = \frac{R}{L} \quad ; \quad R = \frac{L}{Q} \frac{2\pi}{T_0} = 160 \, \Omega$$

Mesures de la bande passante (Measuring the bandwidth)

$$\tau = \frac{2Q}{\omega_0} \quad ; \quad \Delta\omega = \frac{\omega_0}{Q} \quad ; \quad \tau\Delta\omega = 2$$

Vérification « expérimentale » avec AnHarm : (experimental results with « AnHarm »)





- **Lien avec l'optique (analogy with optical systems)**

τ : light comes from independent atoms which emit on time scales of about 10^{-8} seconds.

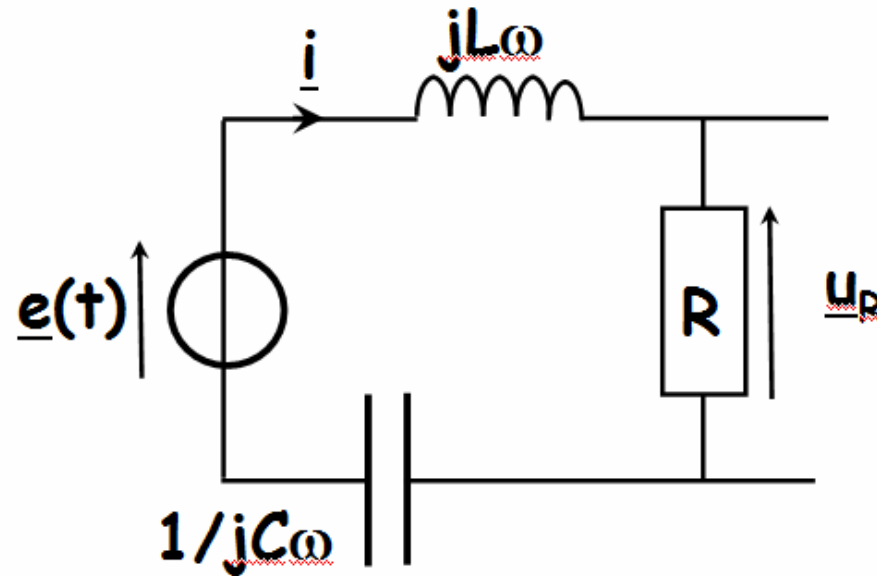
$\Delta\nu$: the frequency of the used light varies from ν_1 to $\nu_1 + \Delta\nu$ (quasi-monochromatic wave)

$$\tau \Delta\nu \approx 1$$

(Compare with : $\tau\Delta\omega = 2$)

b) Régime sinusoïdal forcé : (AC - Alternating current - driven RLC circuits)

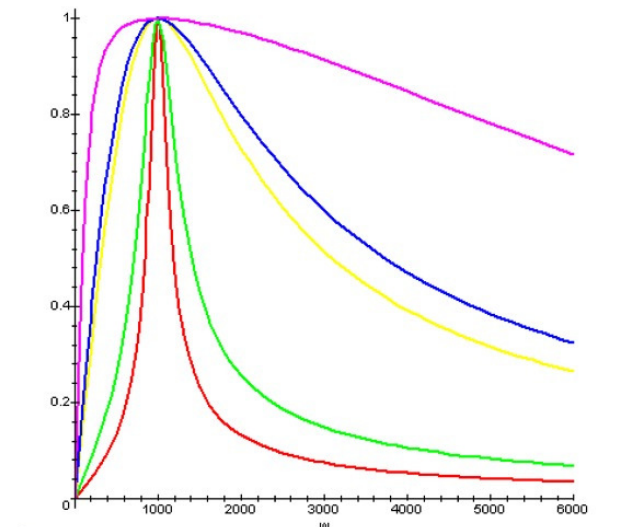
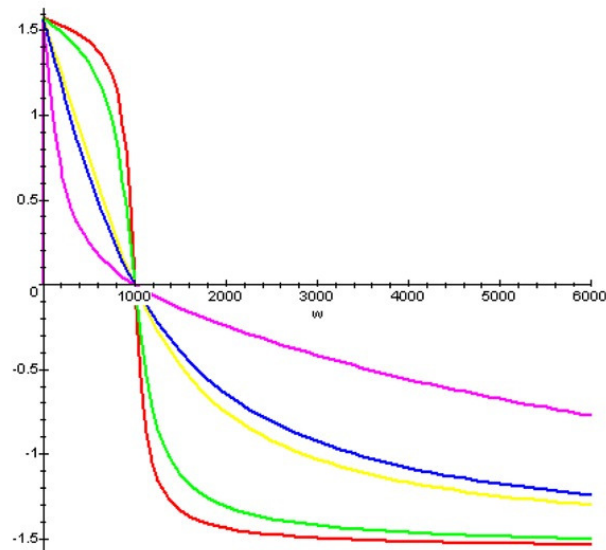
Tension aux bornes de la résistance : (voltage across the resistor)

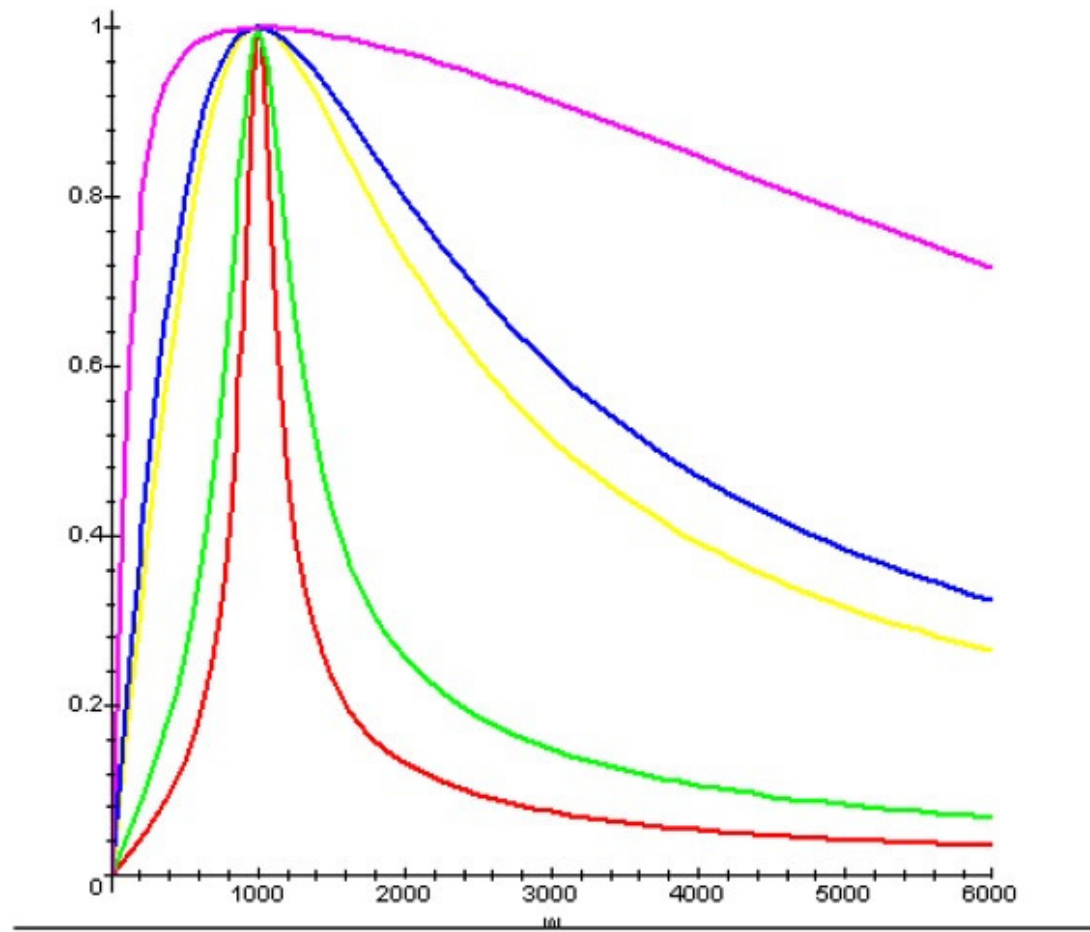


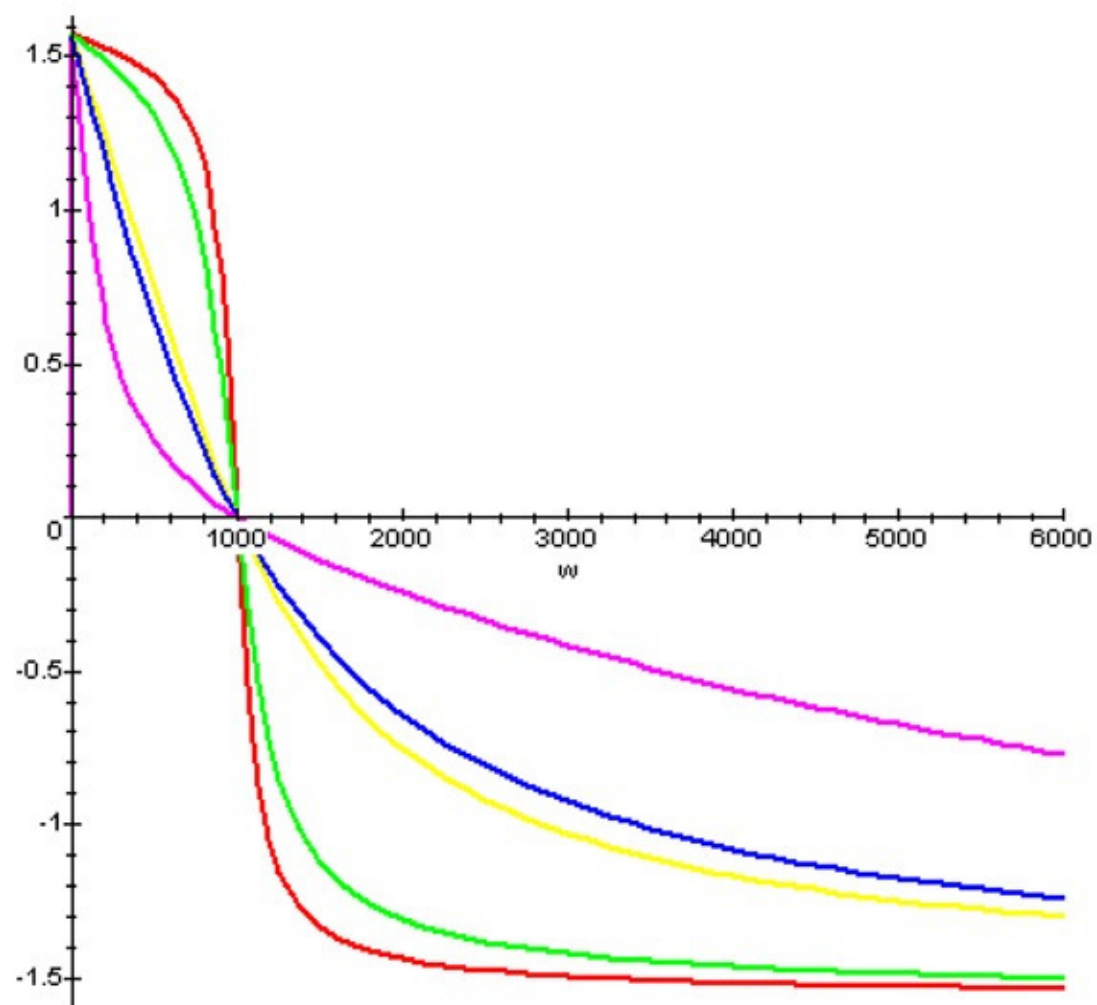
$$\underline{u}_R = \frac{R}{R + jL\omega + \frac{1}{jC\omega}} \underline{e} = \frac{jRC\omega}{1 - LC\omega^2 + jRC\omega} \underline{e}$$

$$U_{R,m} = \frac{R}{\sqrt{R^2 + (L\omega - \frac{1}{C\omega})^2}} E_m$$

$$\tan \phi_R = -\frac{L\omega - \frac{1}{C\omega}}{R} \quad \text{avec} \quad \cos \phi_R > 0 \left(\phi_R \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \right)$$





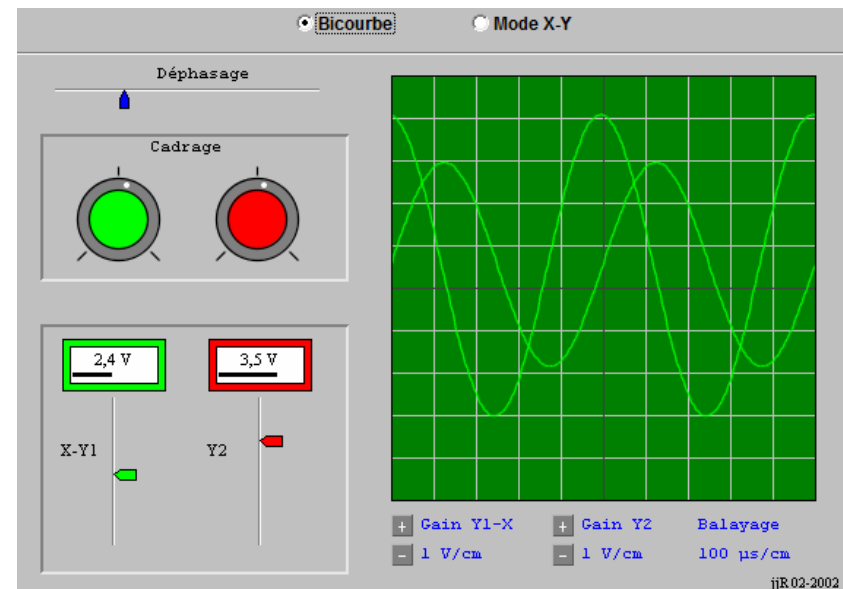
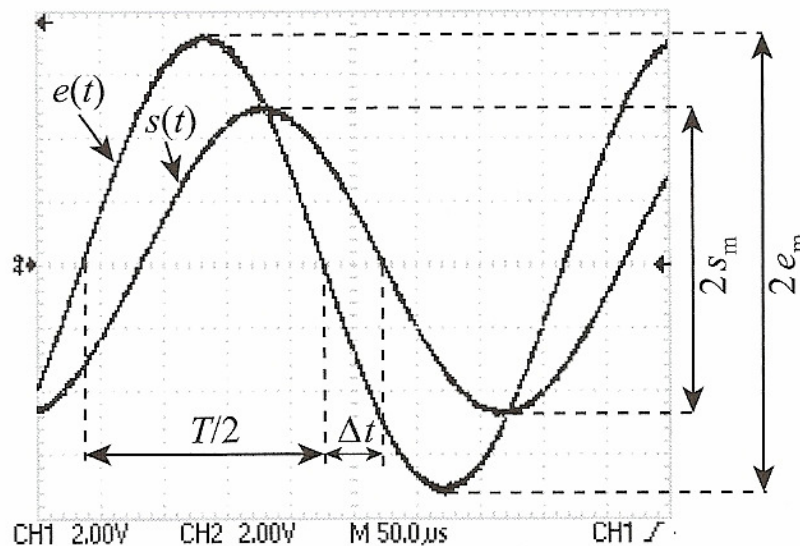


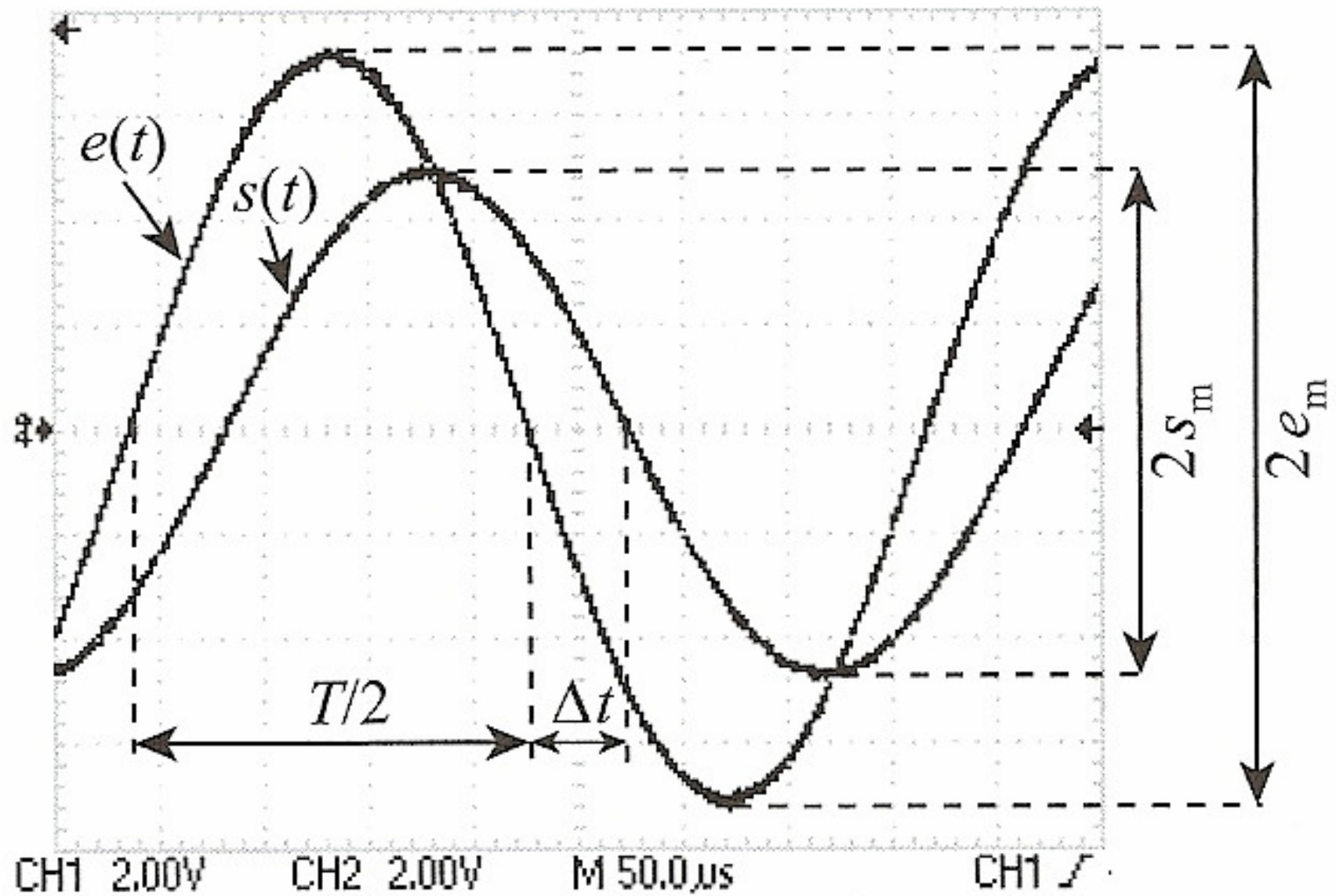
Animations JJR :

[JJR/Electricité/Circuit RLC en régime sinusoïdal b](#)

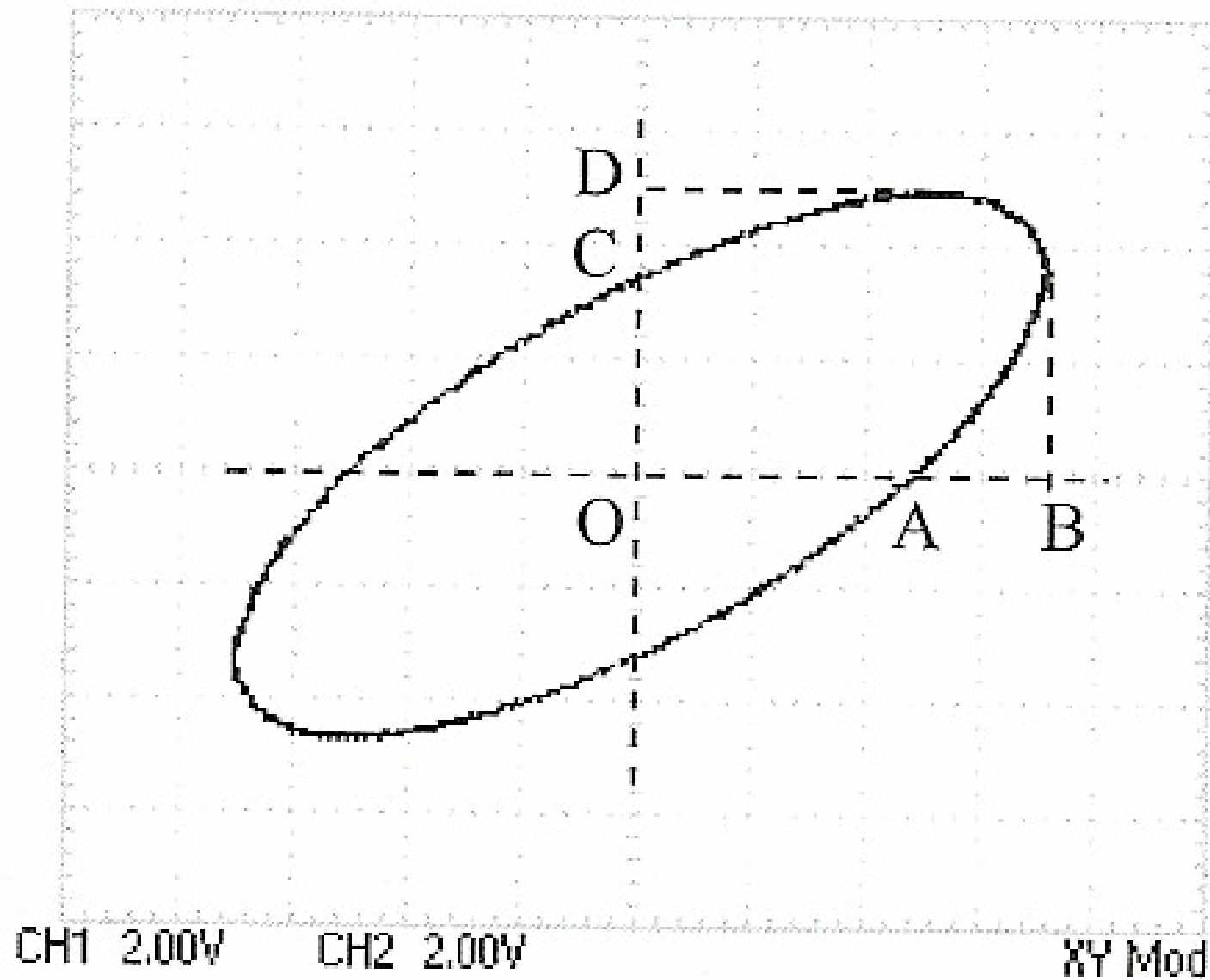
How to measure induced phase change φ ?

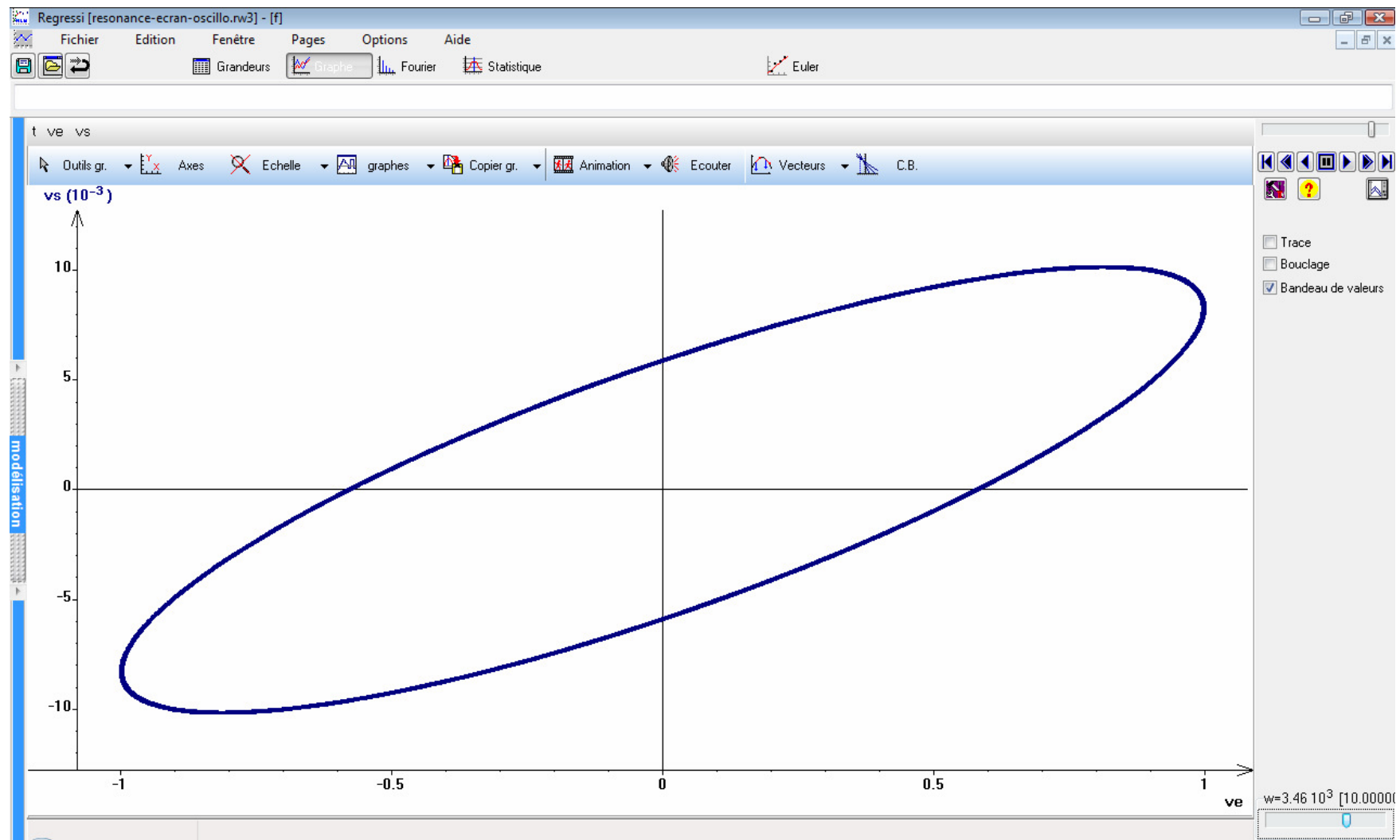
[JJR/Electronique/Mesure de déphasage](#)



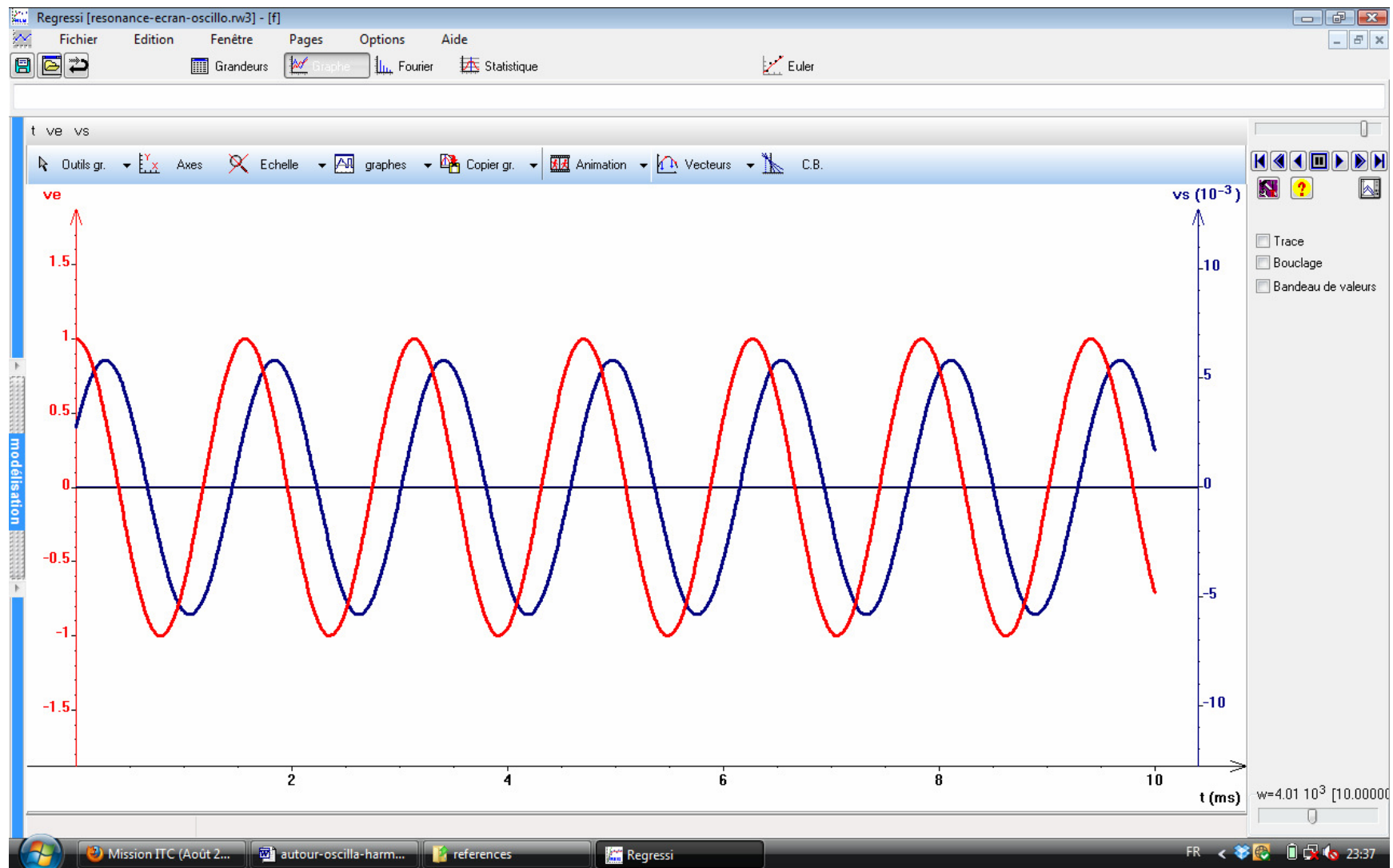


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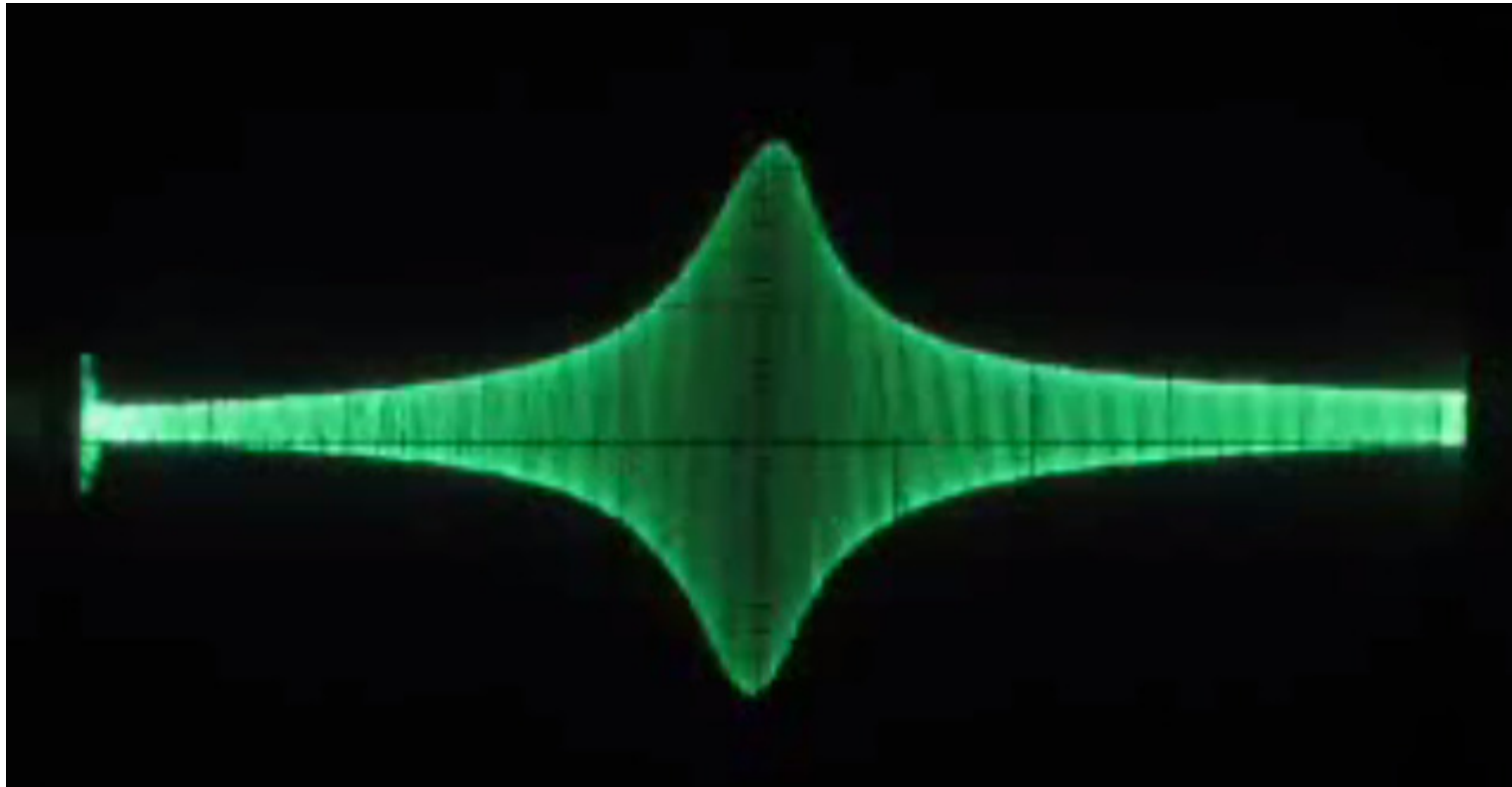


Regressi software : file « [resonance-ecran-oscillo](#) »



Regressi software : file « [resonance-ecran-oscillo](#) »

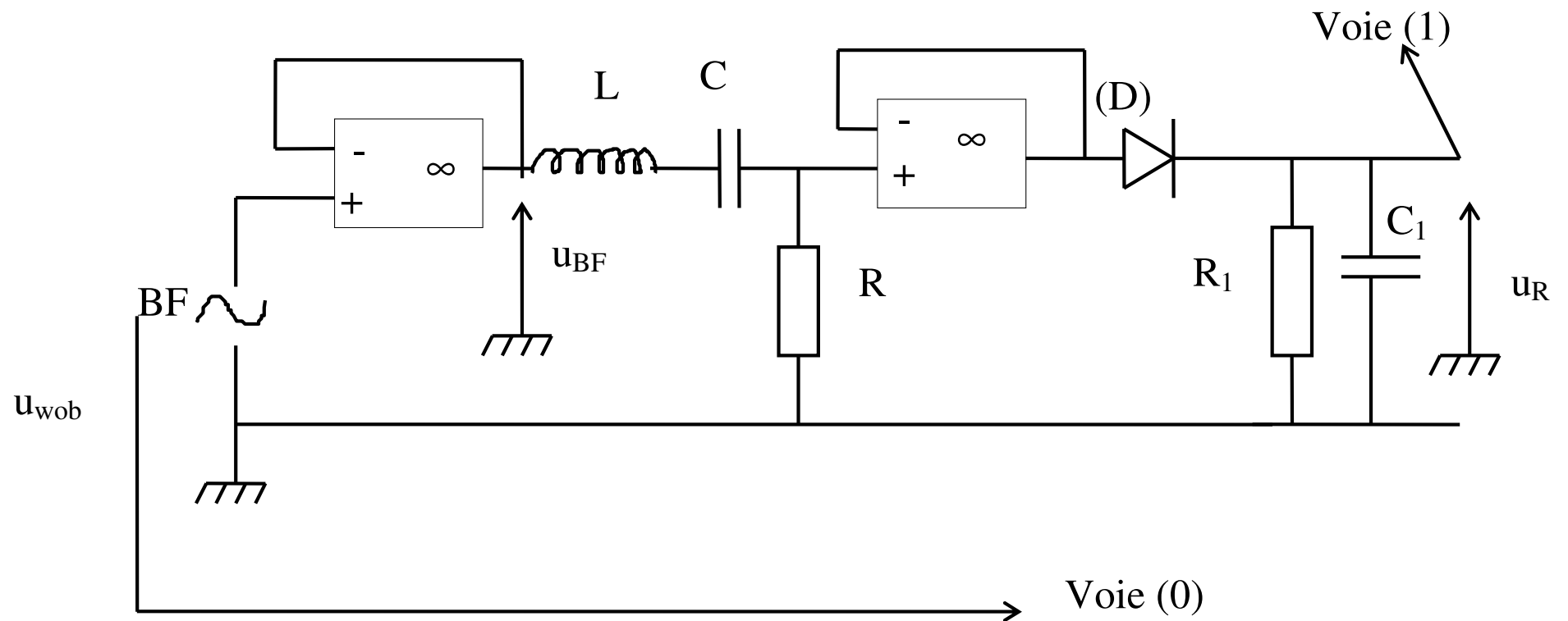
Vidéo : « Résonance dans un circuit RLC, utilisation de la wobulation »



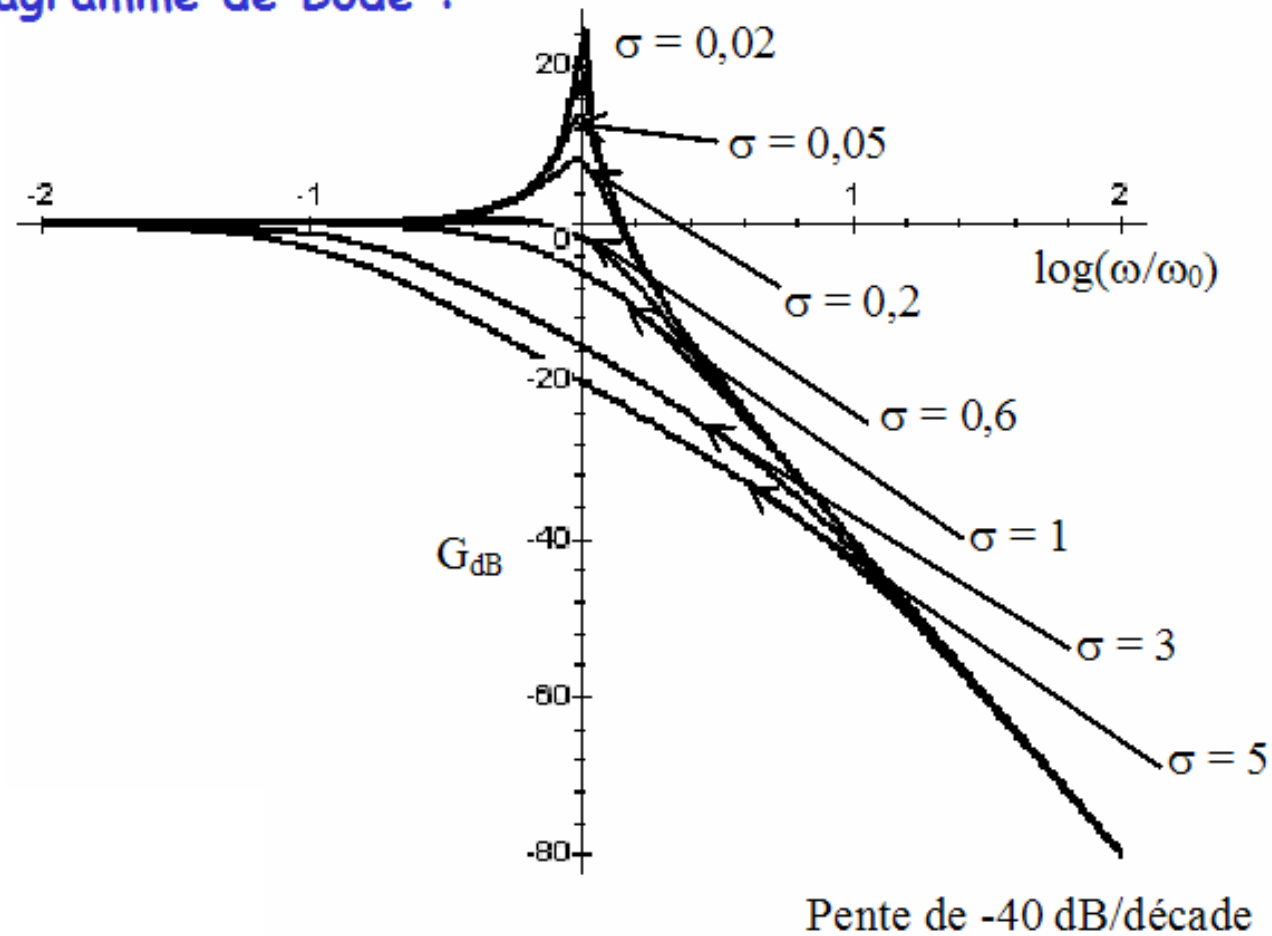
Principe de la wobulation du générateur BF : (operating instructions)

- * Choisir une fréquence minimale égale à $f_{\min} = 2 \text{ kHz}$.
- * Choisir une fréquence maximale égale à $f_{\max} = 20 \text{ kHz}$.
- * Visualiser la tension U_{wob} (prise à l'arrière du générateur BF) : on observe une rampe de tension.
La tension varie de manière linéaire de la valeur $U_1 = 0 \text{ V}$ (correspondant à f_1) jusqu'à la valeur $U_2 \approx 10 \text{ V}$ (correspondant à f_2).
Il existe une relation affine entre la tension de wobulation U_{wob} et la fréquence f de balayage : $f = aU_{\text{wob}} + b$ (déterminer numériquement les deux coefficients a et b).
- * Régler la durée de la rampe de tension à 8 s environ.

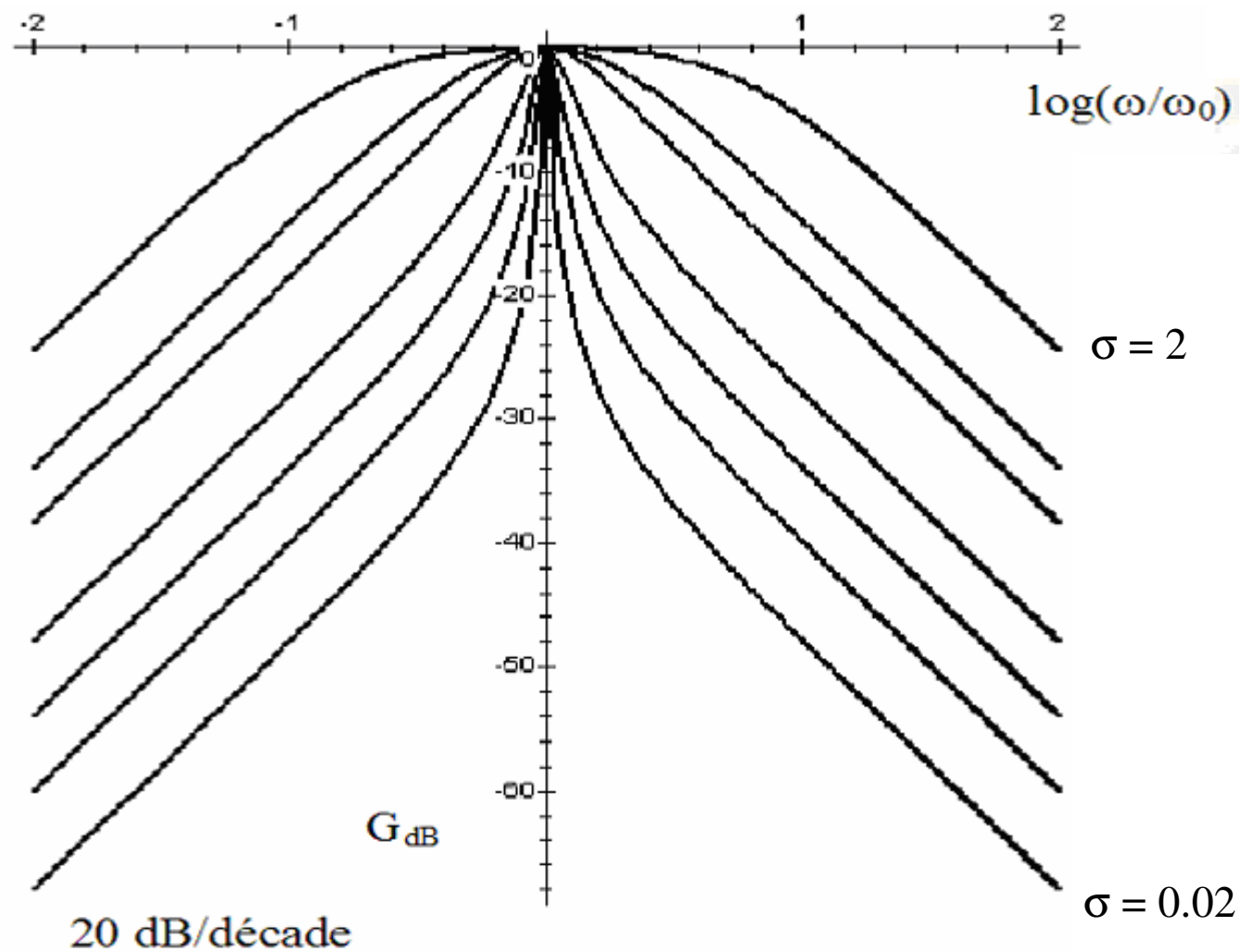
Montage expérimental à réaliser : (experimental setup)



Tracé du diagramme de Bode :



Bode diagram : 2nd order lowpass filter



Bode diagram : 2nd order bandpass filter

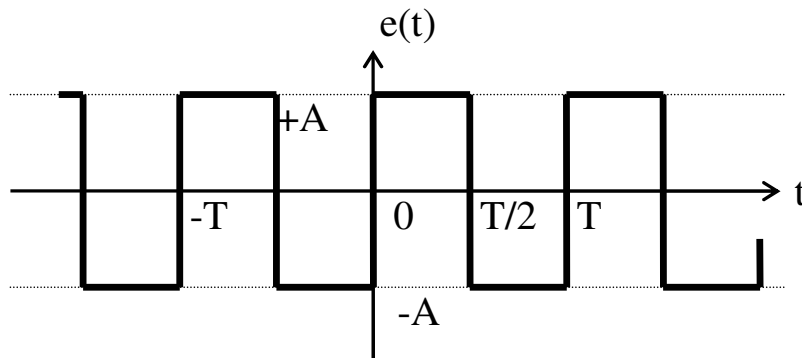
Equivalent systems :

Harmonic oscillators from other areas of engineering are equivalent.

Their mathematical models are identical.

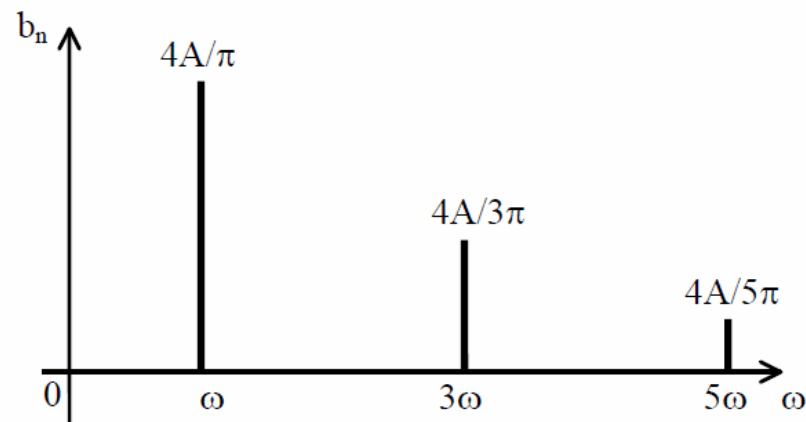
Translational Mechanical	Torsional Mechanical	Series RLC Circuit	Parallel RLC Circuit
Position x	Angle θ	Charge q	Voltage e
Velocity $\frac{dx}{dt}$	Angular velocity $\frac{d\theta}{dt}$	Current $\frac{dq}{dt}$	$\frac{de}{dt}$
Mass M	Moment of inertia I	Inductance L	Capacitance C
Spring constant K	Torsion constant μ	Elastance $1/C$	Susceptance $1/L$
Damping γ	Rotational friction Γ	Resistance R	Conductance $1/R$
Drive force $F(t)$	Drive torque $\tau(t)$	e	di/dt
Undamped resonant frequency f_n :			
$\frac{1}{2\pi} \sqrt{\frac{K}{M}}$	$\frac{1}{2\pi} \sqrt{\frac{\mu}{I}}$	$\frac{1}{2\pi} \sqrt{\frac{1}{LC}}$	$\frac{1}{2\pi} \sqrt{\frac{1}{LC}}$
Differential equation:			
$M\ddot{x} + \gamma\dot{x} + Kx = F$	$I\ddot{\theta} + \Gamma\dot{\theta} + \mu\theta = \tau$	$L\ddot{q} + R\dot{q} + q/C = e$	$C\ddot{e} + \dot{e}/R + e/L = i$

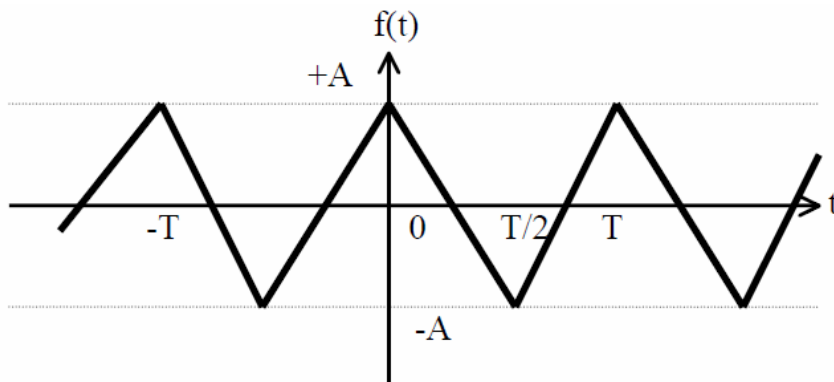
c) Justification de l'analyse harmonique : (Harmonic analysis : functions or signals represented as the superposition of basic sine waves, Fourier analysis)



(Square signal)

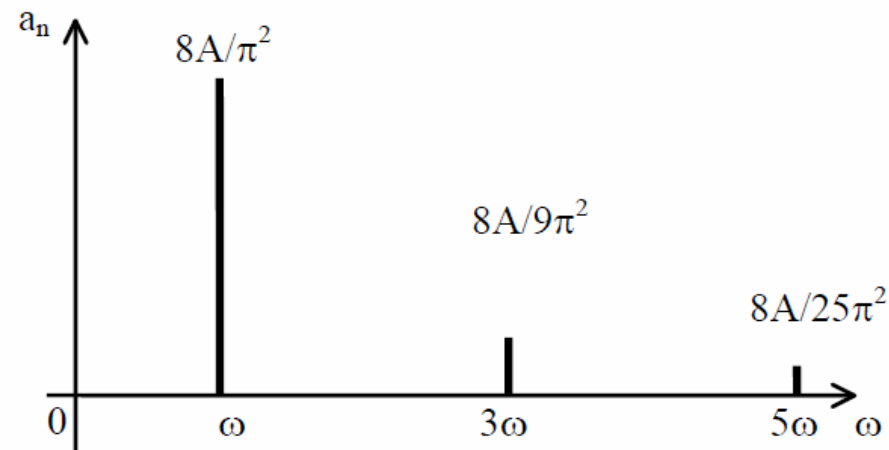
$$e(t) = \frac{4A}{\pi} \left[\sin \omega t + \frac{1}{3} \sin 3\omega t + \frac{1}{5} \sin 5\omega t + \dots \right]$$

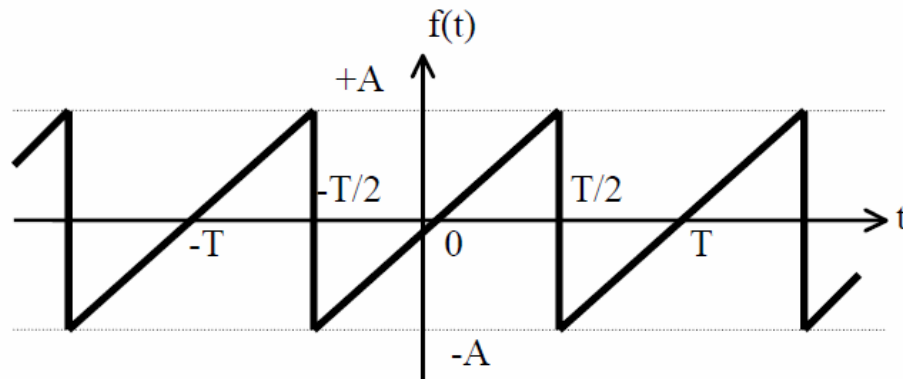




(Triangle signal)

$$f(t) = \frac{8A}{\pi^2} \left[\cos \omega t + \frac{1}{3^2} \cos 3\omega t + \frac{1}{5^2} \cos 5\omega t + \dots \right]$$

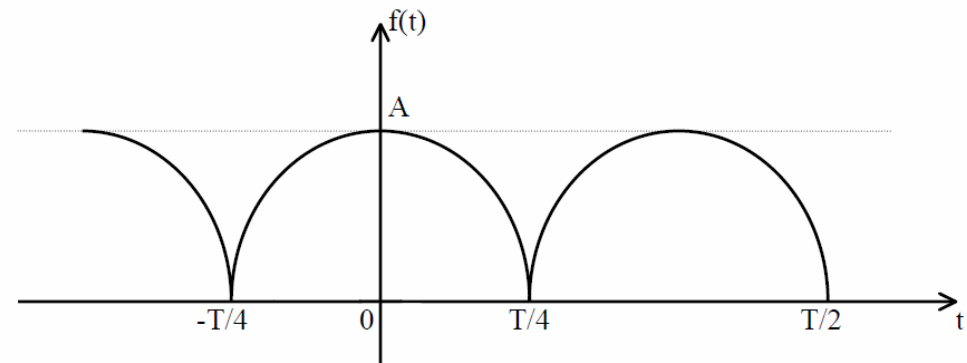




(Sawtooth signal)

(It is named a sawtooth based on its resemblance to the teeth on the blade of a saw)

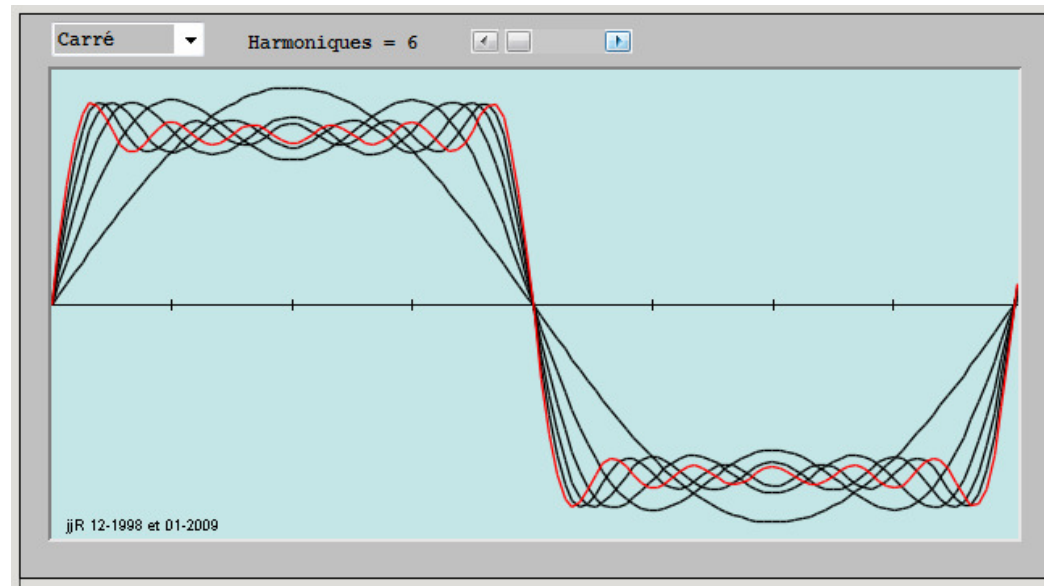
$$f(t) = \frac{2A}{\pi} \left[\sin \omega t - \frac{1}{2} \sin 2\omega t + \frac{1}{3} \sin 3\omega t - \frac{1}{4} \sin 4\omega t + \dots \right]$$



$$f(t) = \frac{2A}{\pi} + \frac{4A}{\pi} \left[\frac{1}{3} \cos 2\omega t - \frac{1}{3.5} \cos 4\omega t + \frac{1}{5.7} \cos 6\omega t + \dots \right]$$

Animations JJR :

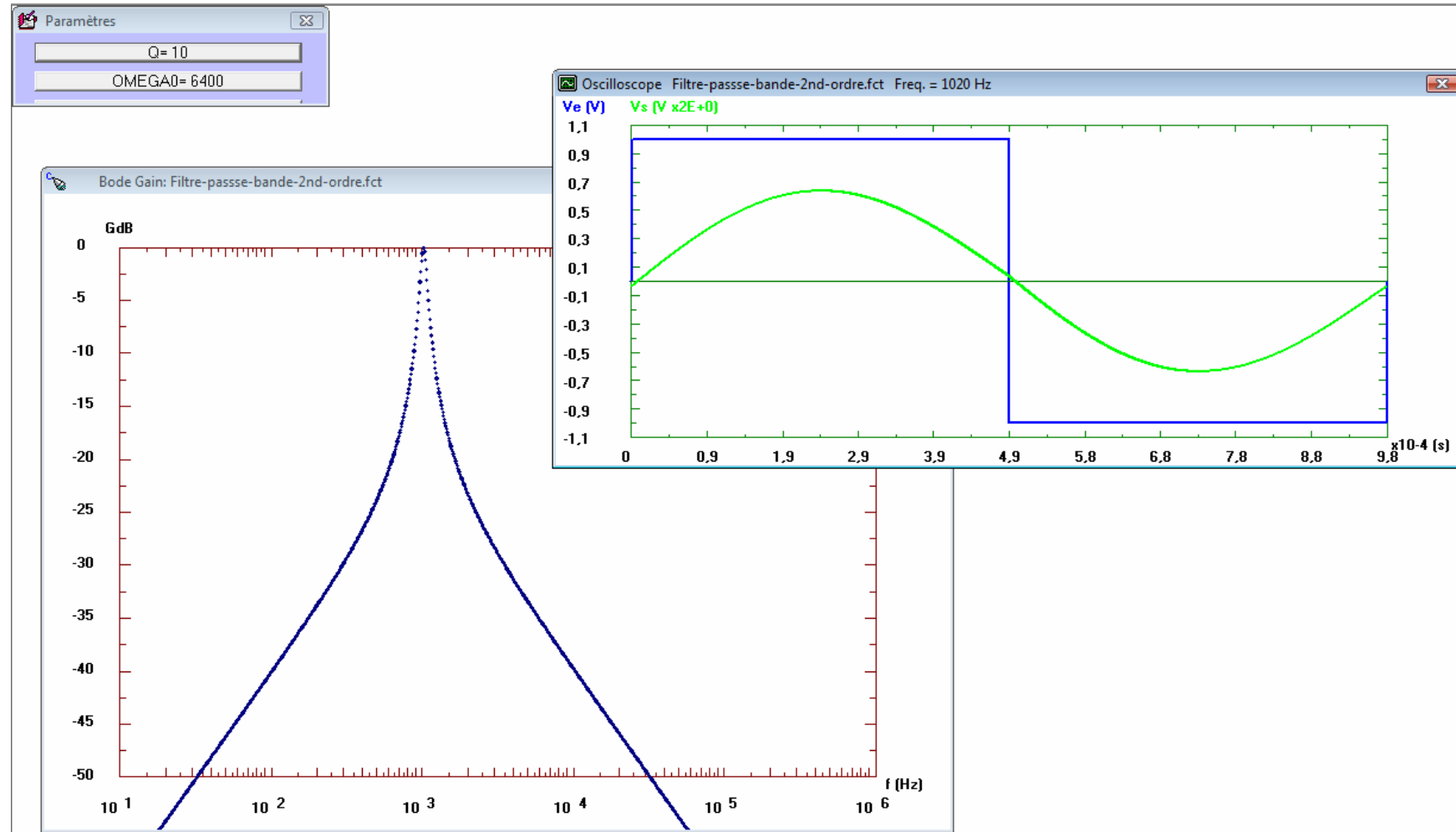
[JJR/Sujets divers/Autres sujets/Séries de Fourier 2](#)

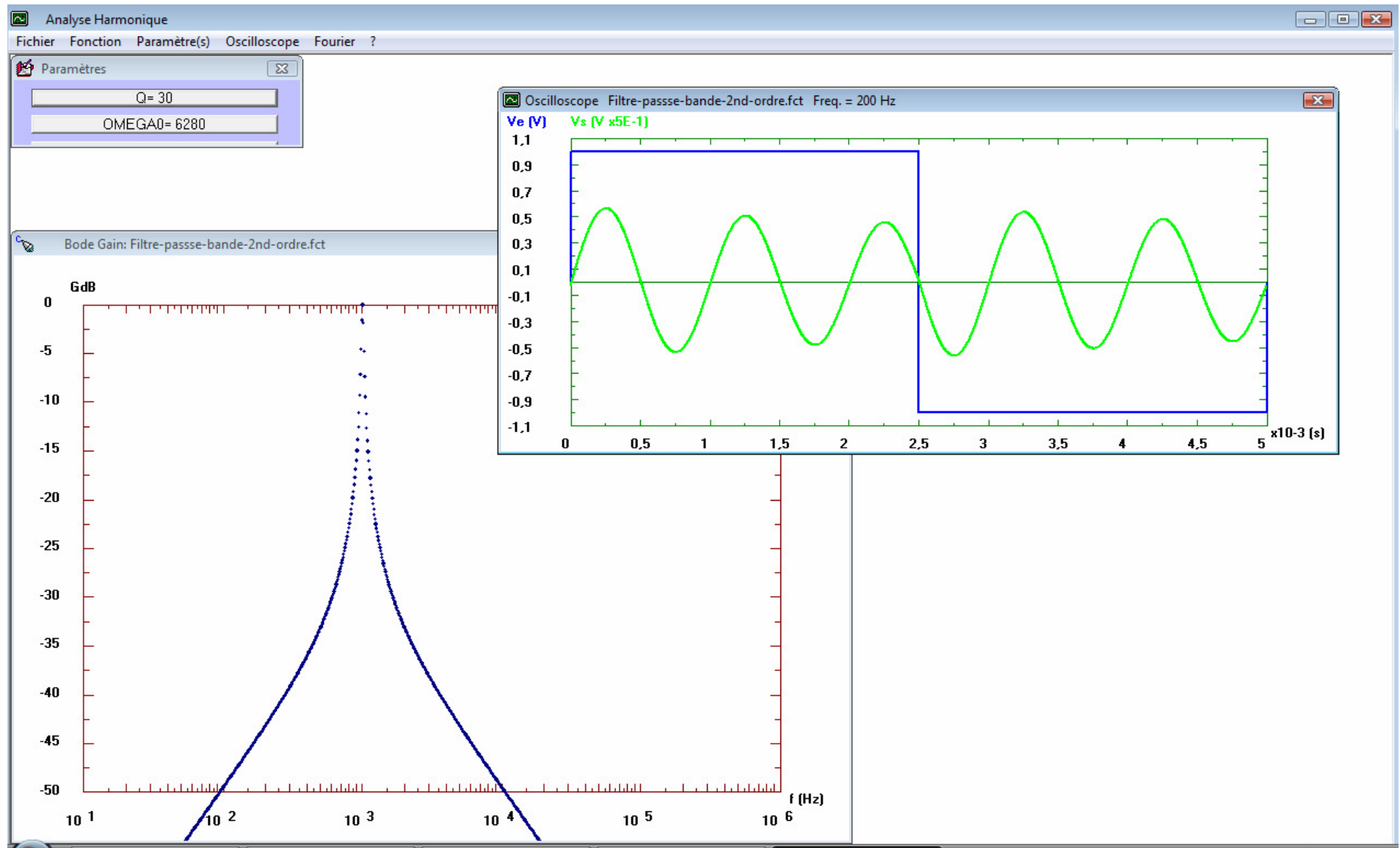


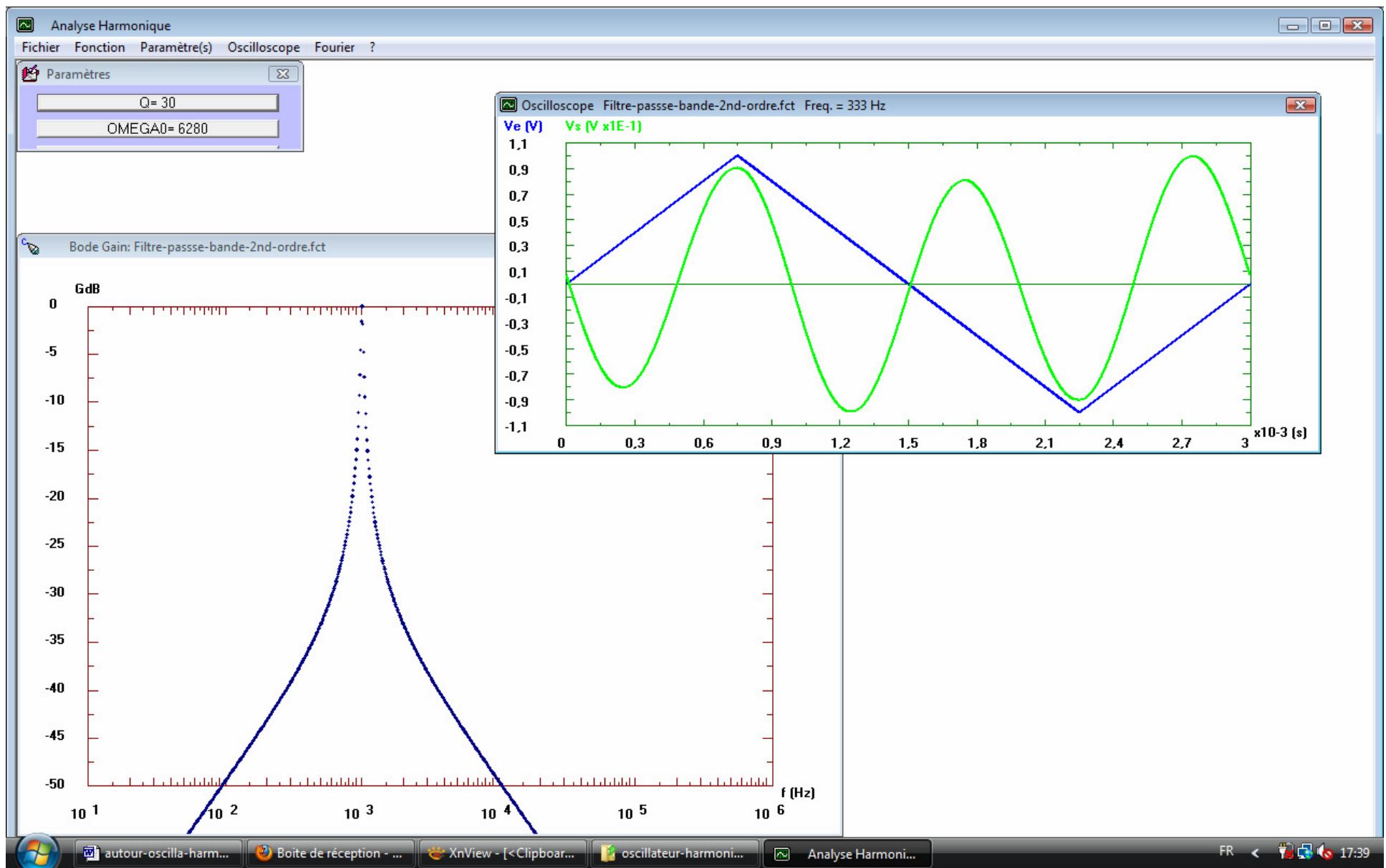
[JJR/Sujets divers/Autres sujets/Synthèse](#)

Utilisation du logiciel AnHarm : (Using the AnHarm software)

Filtre passe bande et signal carré en entrée : (Bandpass filter powered by a square signal)



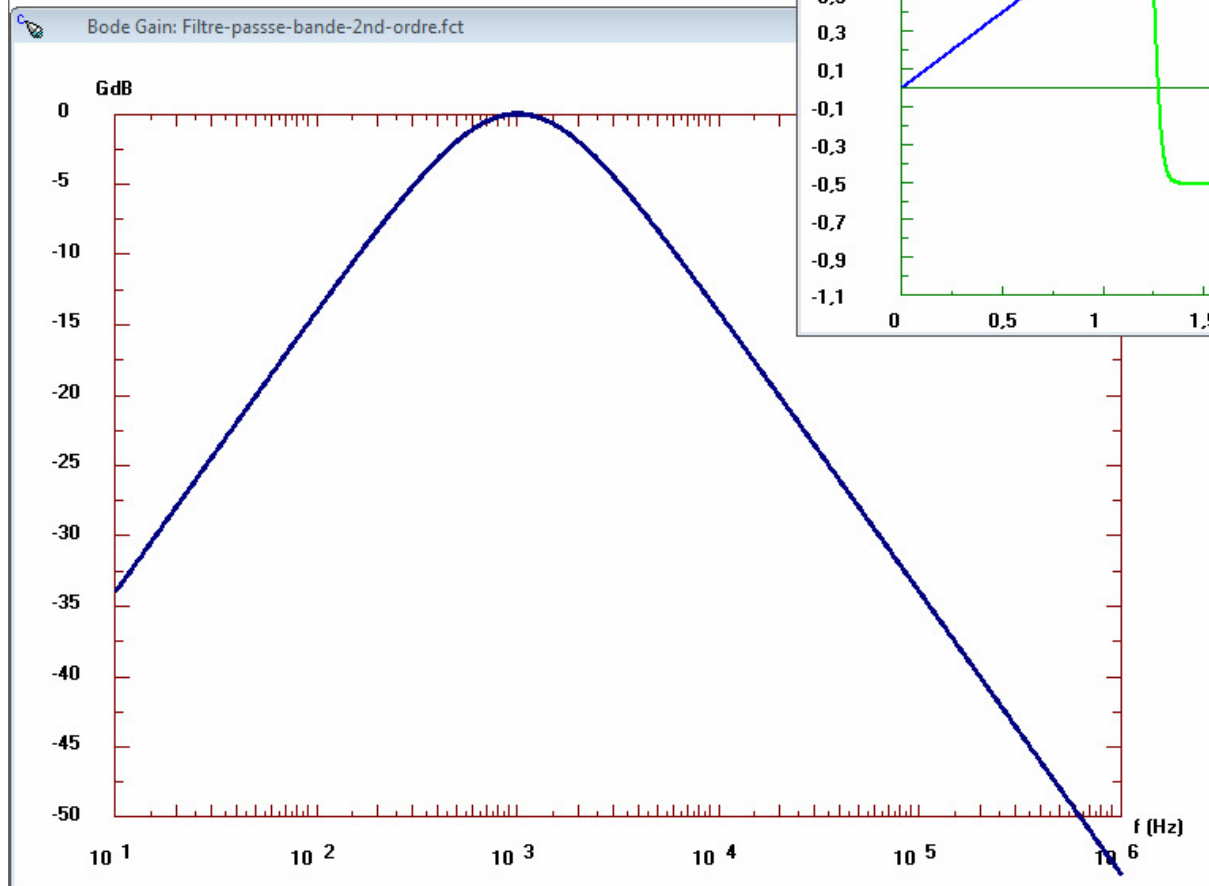


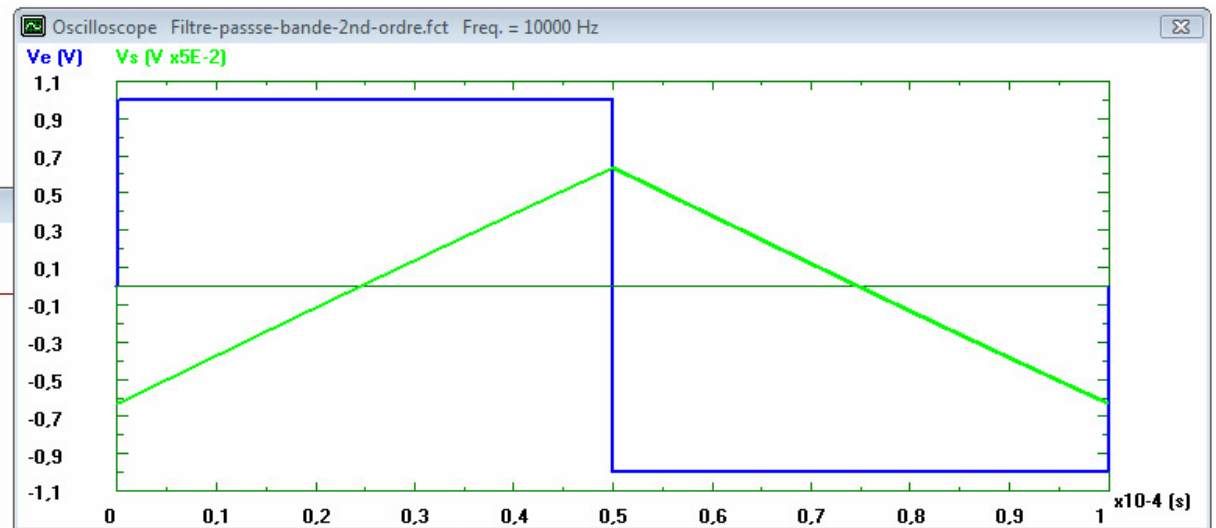
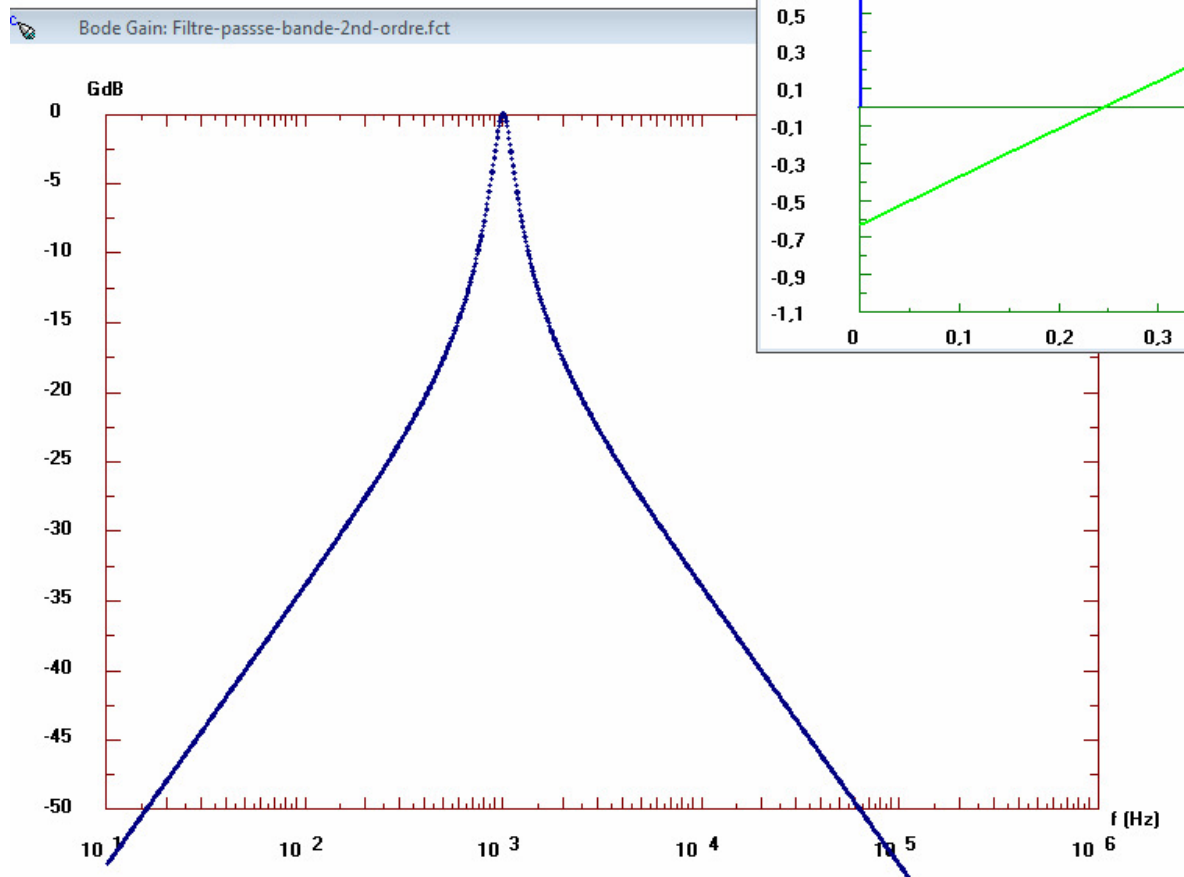
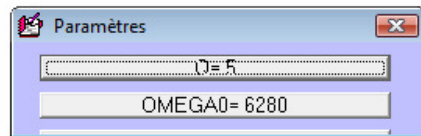


Paramètres

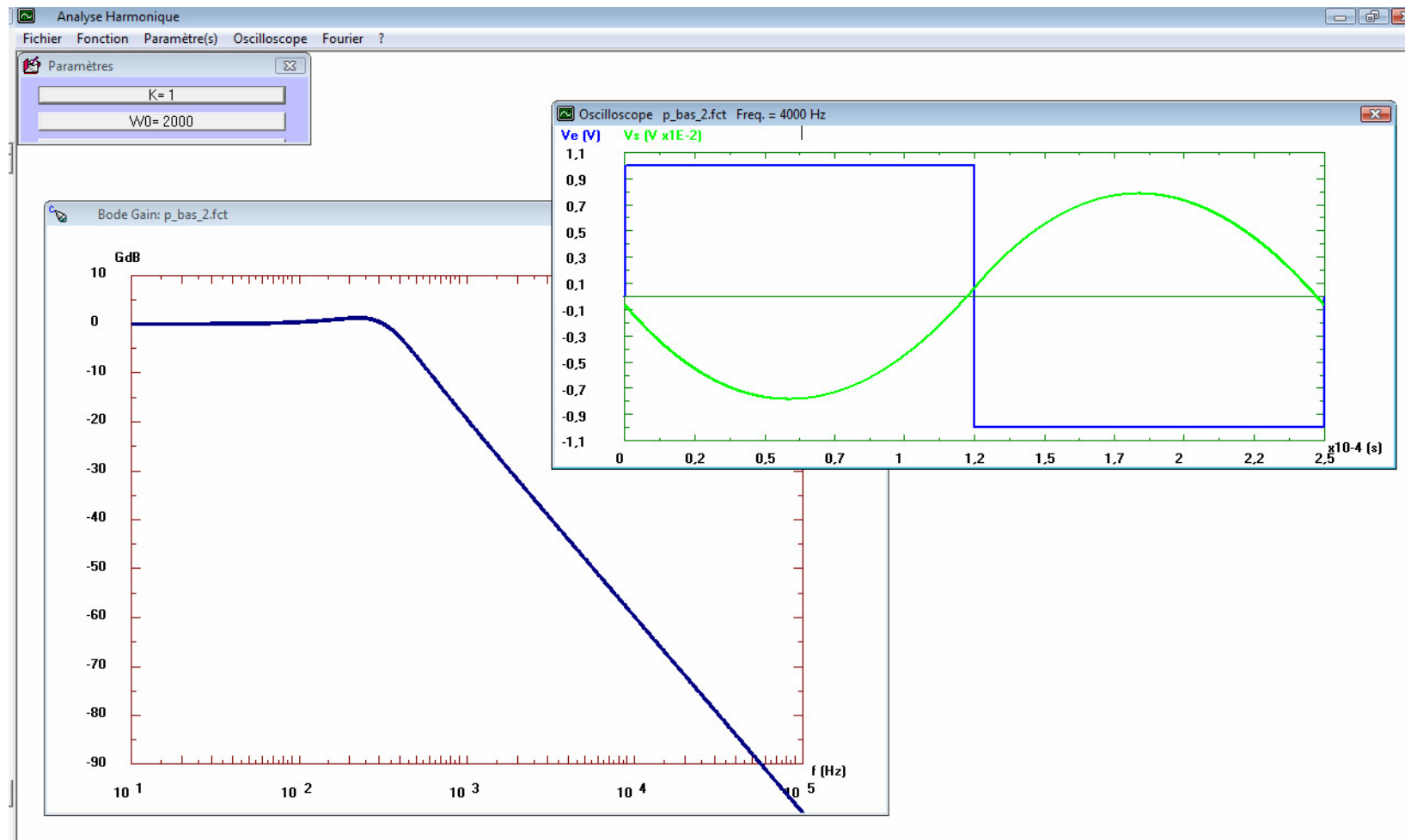
$C_1 = 0.5$

$\text{OMEGA0} = 6280$





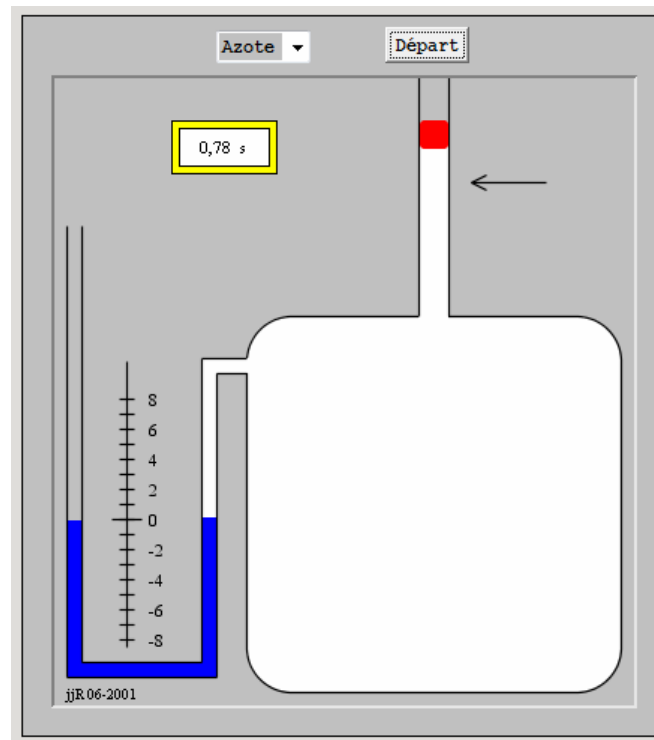
Lowpass filter powered by a square signal with a frequency far from the cutoff frequency ($f \gg f_c$) :

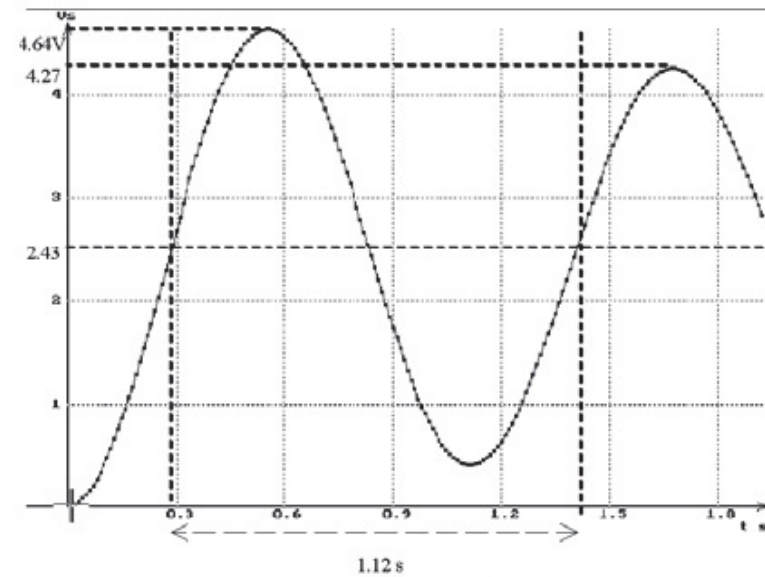
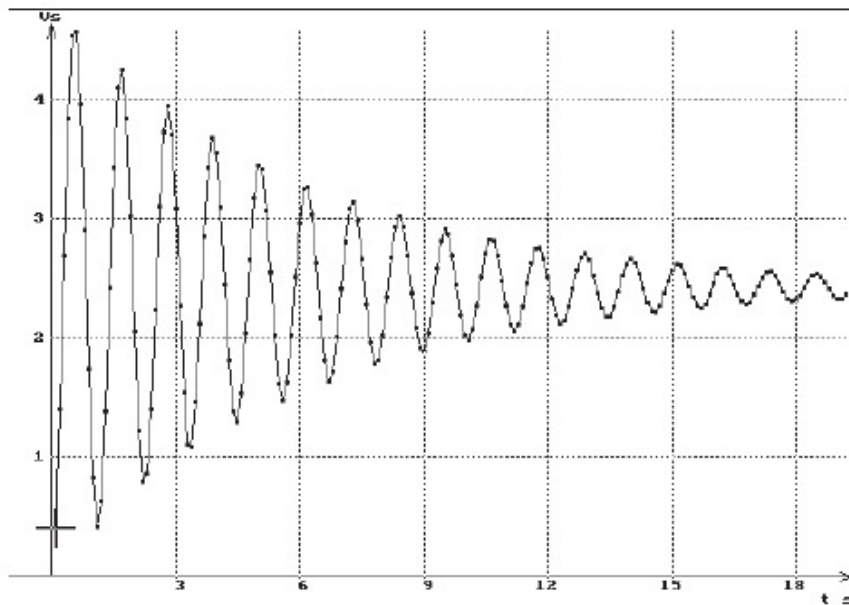
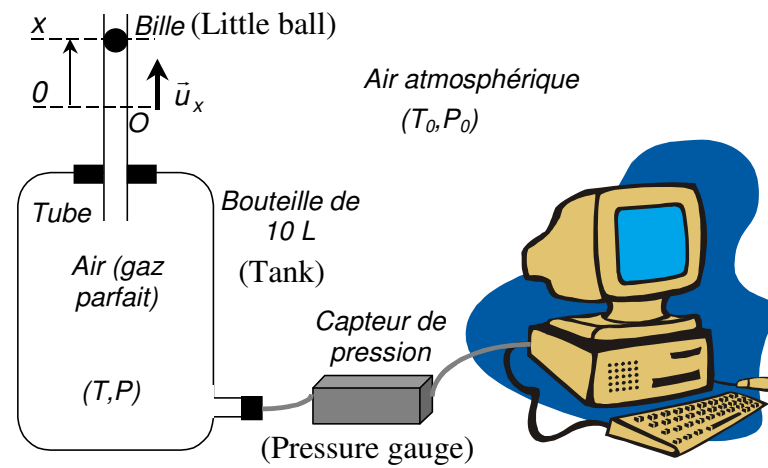


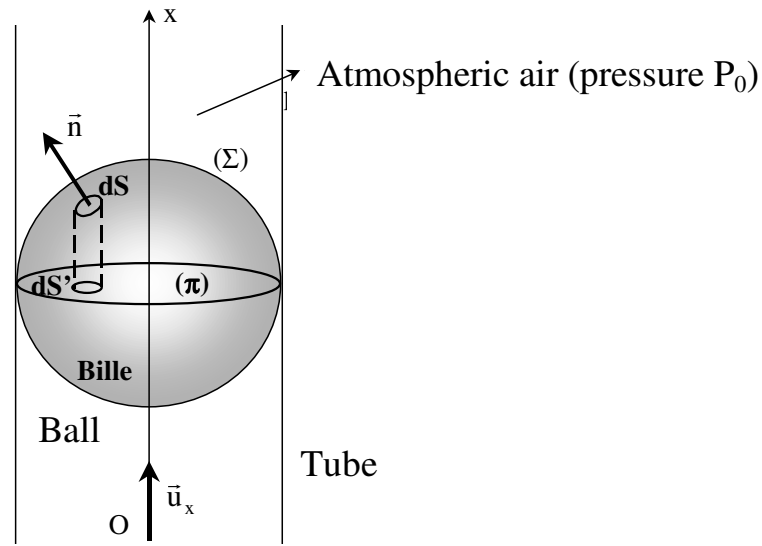
d) D'autres systèmes oscillants : (Other oscillating systems)

Expérience de Rüchardt : animation JJR

[JJR/Thermodynamique/Mesure de gamma a](#)







$$m\ddot{x} = s(P - P_{\text{eq}})$$

$$\frac{dP}{P} + \gamma \frac{dV}{V} = 0 \quad (\text{From gamma law}) \quad ;$$

$$P - P_{\text{eq}} = -\gamma \frac{s x}{V_0} P_{\text{eq}}$$

$$\ddot{x} = -\gamma \frac{s^2 P_{\text{eq}}}{m V_0} x$$

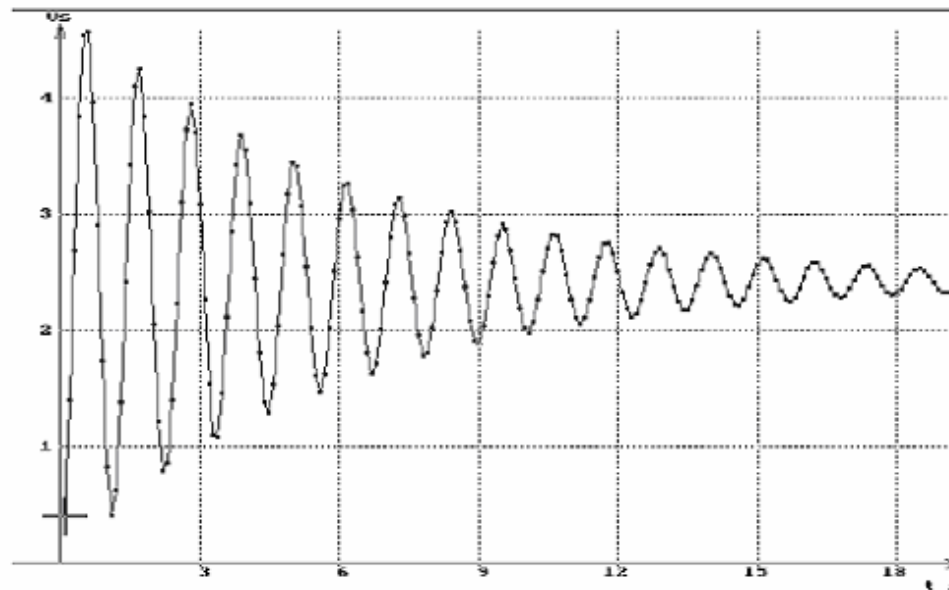
$$\tau = 2\pi / \omega_0 = 2\pi \sqrt{m V_0 / \gamma s^2 P_{\text{eq}}}$$

Etude énergétique : (energetic study)

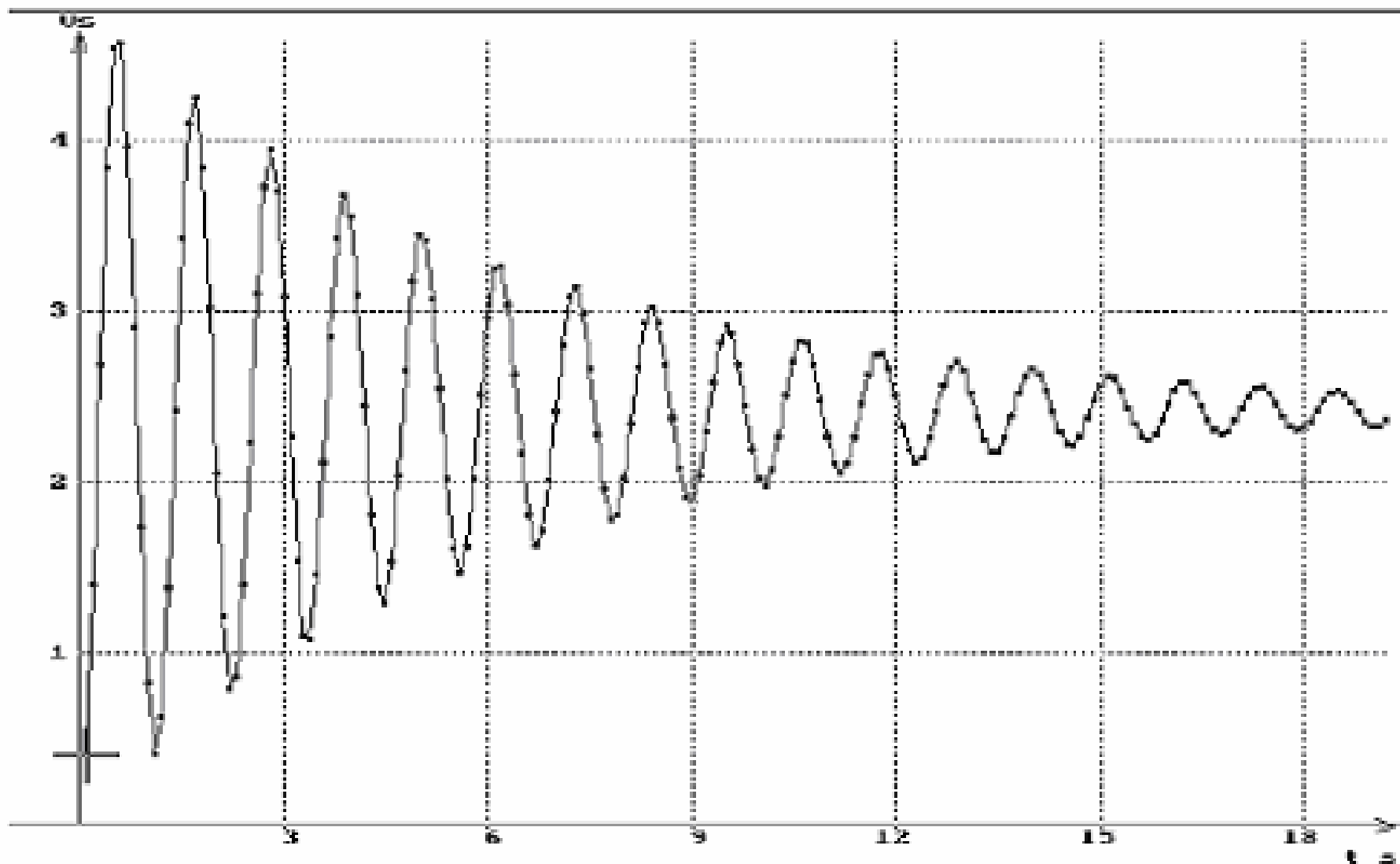
1^{ère} explication : force de frottement fluide (viscous damping, Stokes's law) :

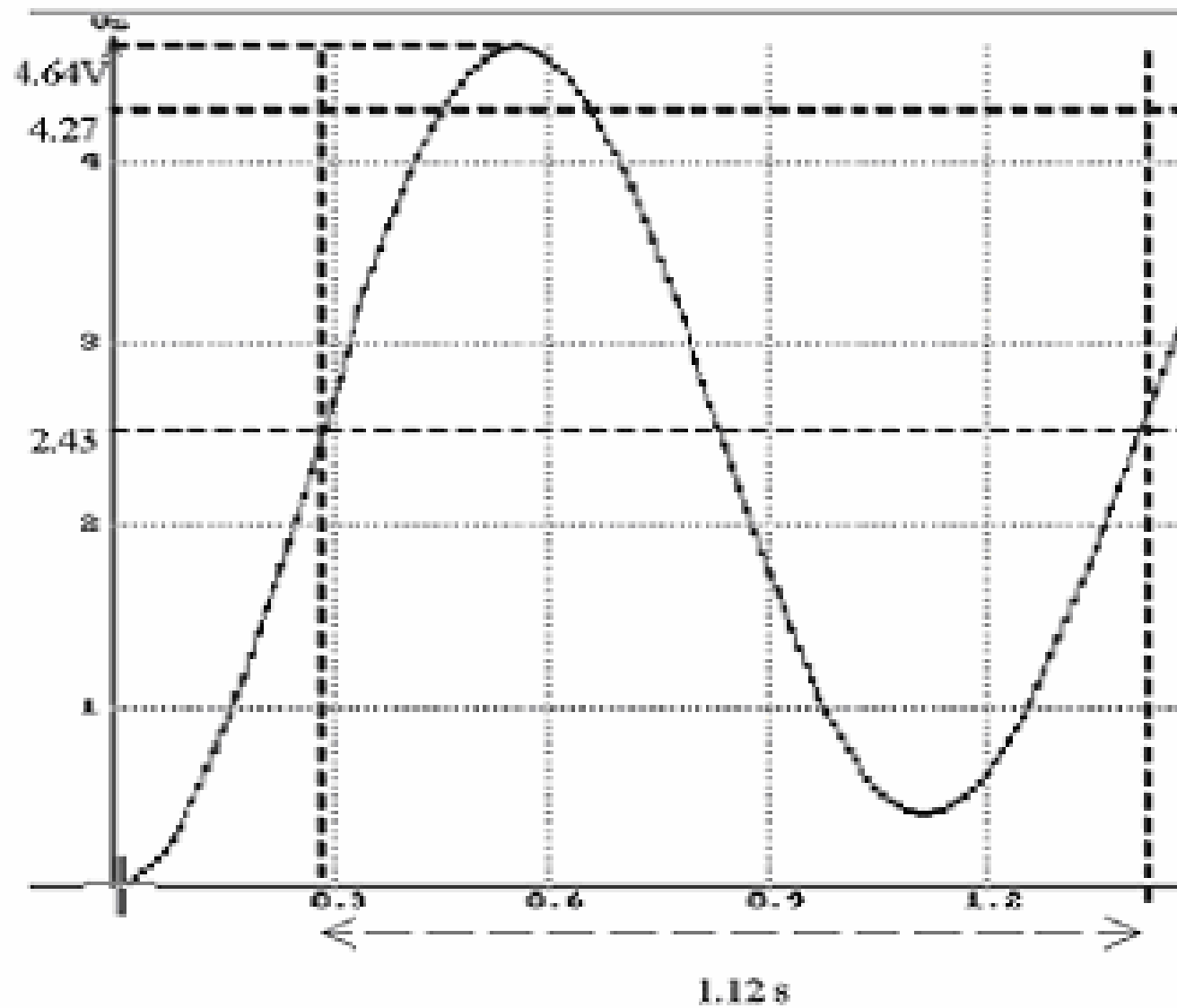
$$\vec{f} = -6\pi\eta r \vec{v} \quad ; \quad \frac{d^2x}{dt^2} + 2\lambda \frac{dx}{dt} + \omega_0^2 x = 0$$

$$u_s = u_s^0 + Ae^{-\lambda t} \cos(\omega t + \varphi) \quad \left(\lambda = \frac{6\pi\eta r}{m} \right)$$



The Stokes's law is not relevant here because the air does not go around the ball.





2^{ème} explication : échanges thermiques avec la paroi (heat transfers with the tube)

Système : la bille et le gaz (the ball and the gas)

Etat initial : l'instant où on lâche la bille (when the ball is dropped)

Etat final : équilibre de la bille (equilibrium position)

- Variation d'énergie interne du gaz : (Variation of internal energy)

$$\Delta U = n \frac{R}{\gamma - 1} (T_f - T_i) = 0 \quad (T_i = T_f)$$

- Variation de l'énergie cinétique de la bille : (Variation of kinetic energy)

$$\Delta E_c = 0 \quad (v_i = v_f = 0)$$

- La variation d'énergie potentielle de la bille : (Variation of gravity potential energy)

$$\Delta E_p = -mgx_0$$

- Le travail des forces de pression : (Work done by pressure forces)

$$W = P_0 s x_0$$

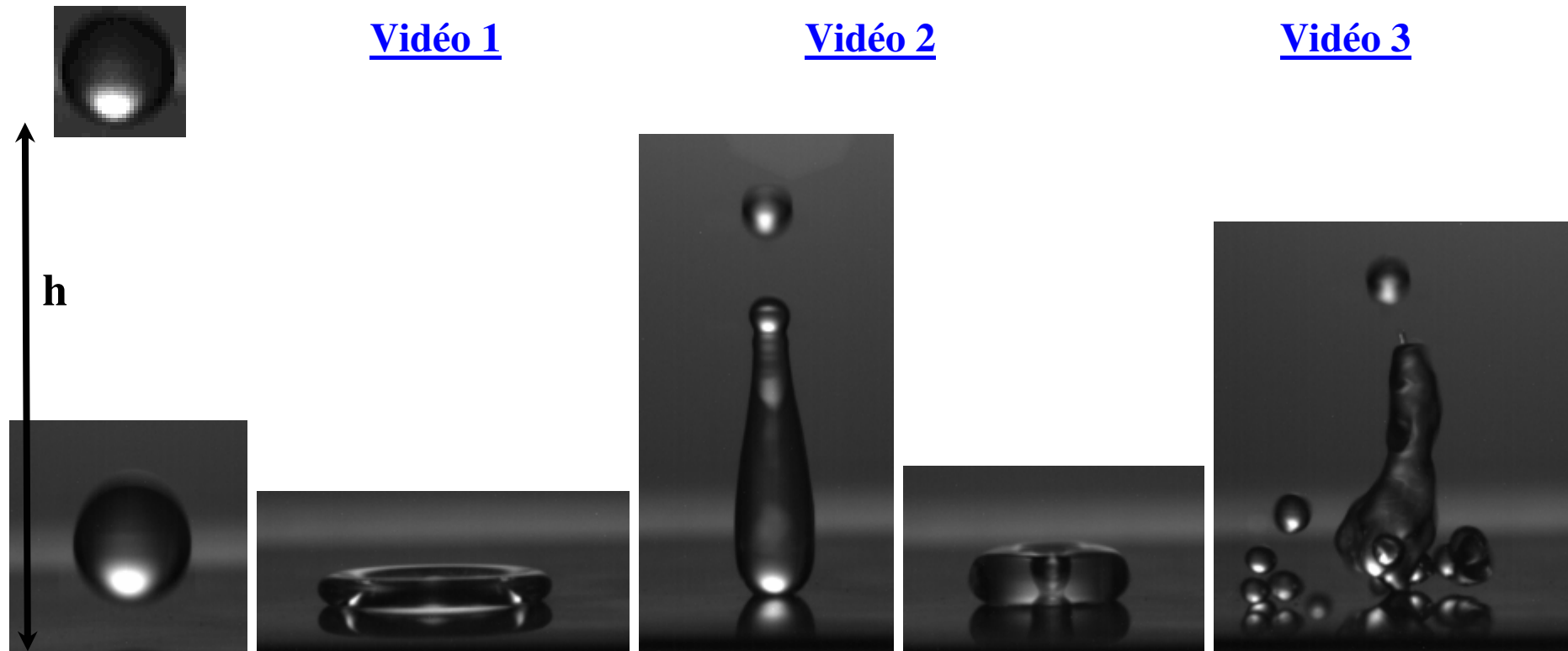
- Le transfert thermique : (heat transfer with the tube)

$$Q = \Delta U + \Delta E_c + \Delta E_p - W = -(mg + P_0 s)x_0 < 0$$

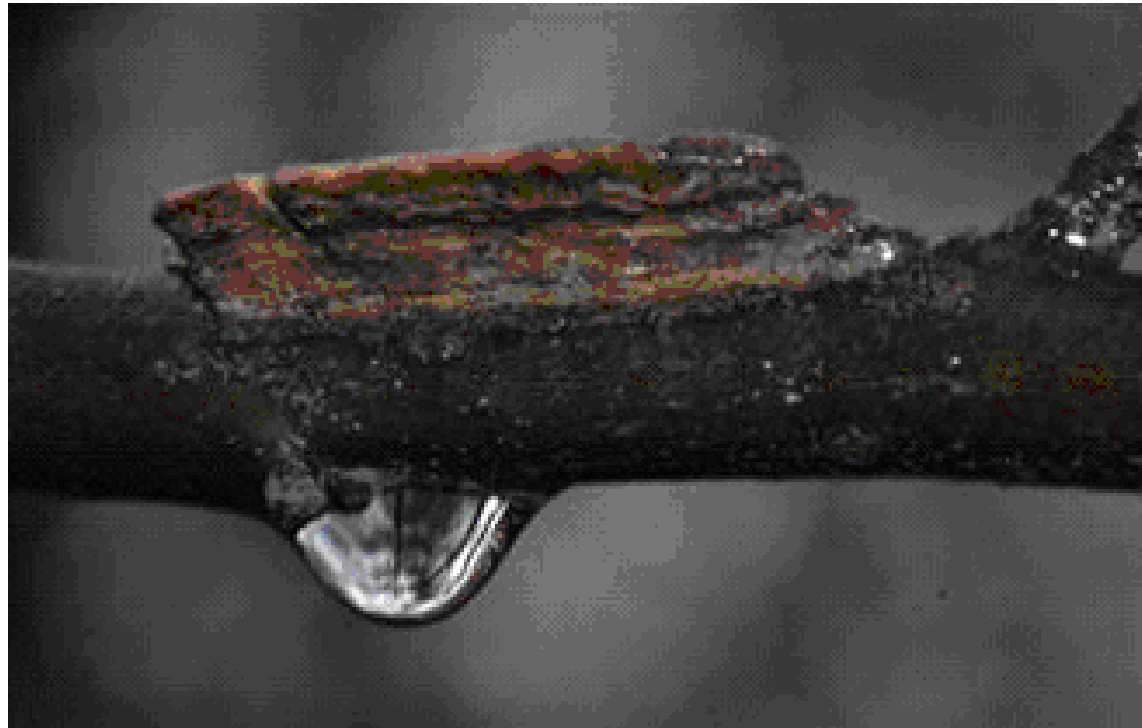
Le gaz donne de la chaleur à l'extérieur (la paroi).

The gas gives heat on the outside (the tube)

Vidéo des chutes de gouttes d'eau : (free falls of water droplets)



Impact of a water droplet on a super – hydrophobic surface



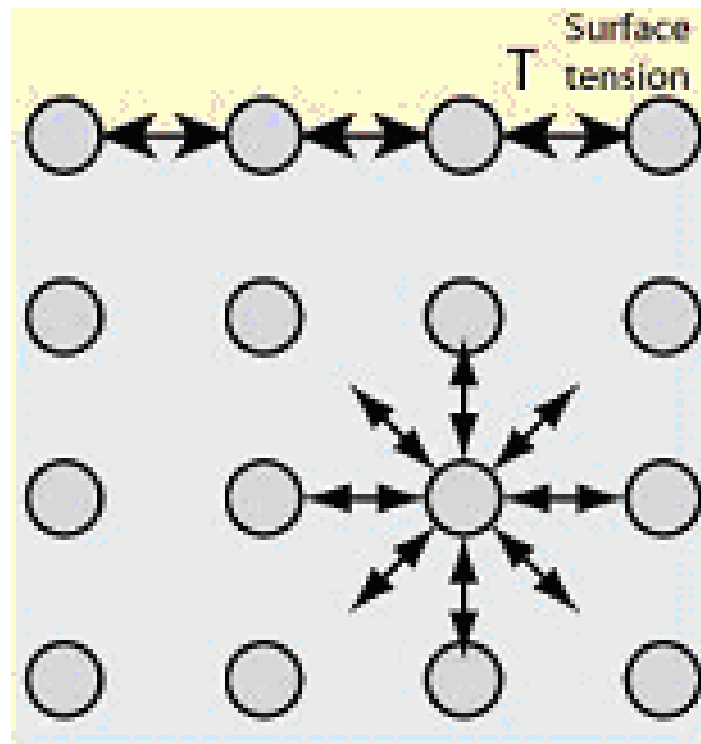
Surface tension and adhesion determine the shape of this drop on a twig.

It dropped a short time later, and took a more nearly spherical shape as it fell.

Falling drops take a variety of shapes due to oscillation and the effects of air friction.

The cohesive forces between molecules down into a liquid are shared with all neighboring atoms.

Those on the surface have no neighboring atoms above and exhibit stronger attractive forces upon their nearest neighbors on the surface.



This enhancement of the intermolecular attractive forces at the surface is called surface tension

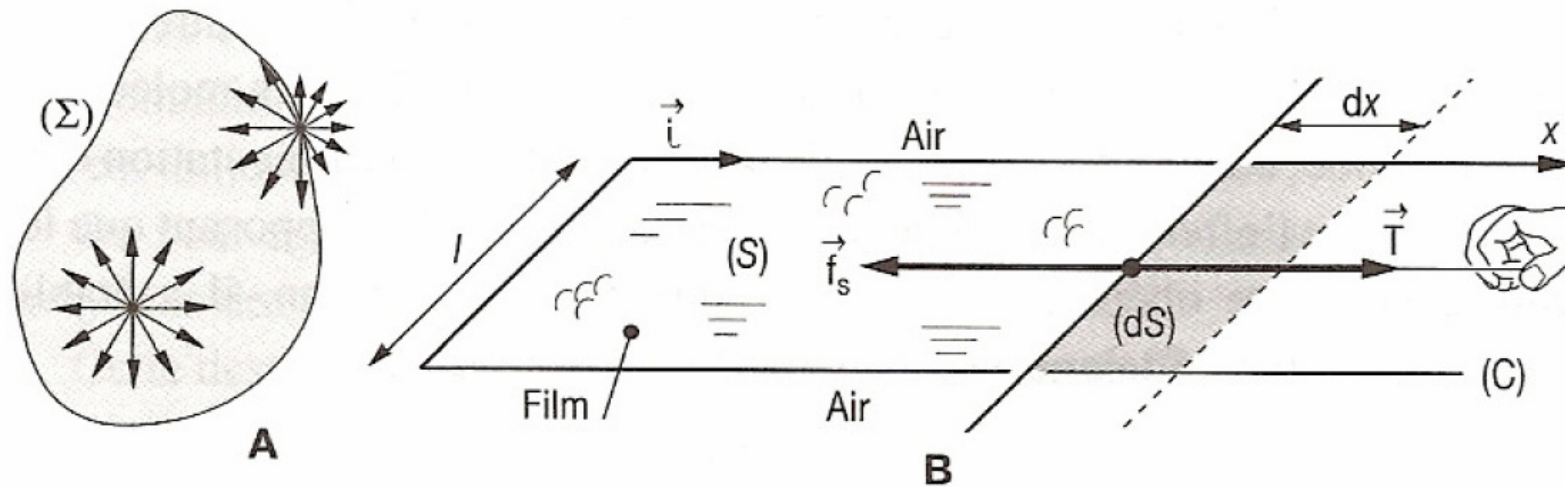
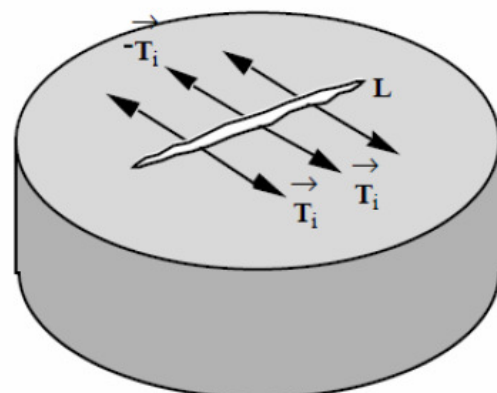


Figure A : interactions de Van der Waals au sein du fluide et sur son interface.

Figure B : forces interfaciales sur un film de savon.

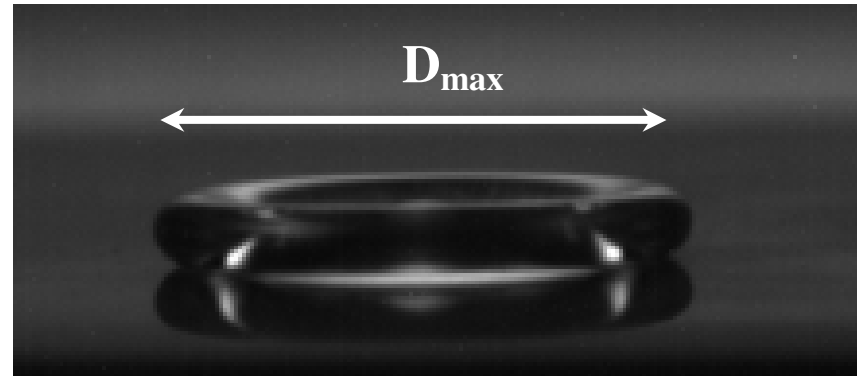
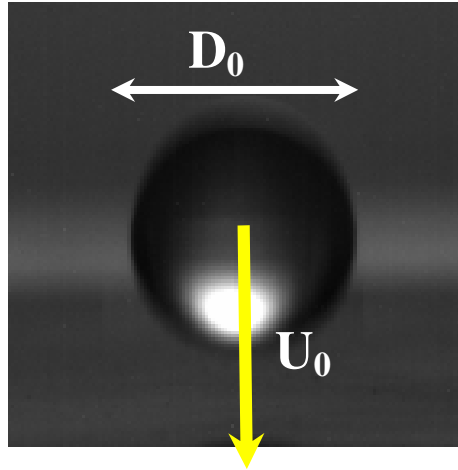


Fente L de longueur l ; tractions T

$$\gamma = T/L : \text{ tension superficielle (unité: N.m}^{-1}\text{)}$$

Maximal deformation of the drop :

The water droplet will be dropped from different heights using a burette



D_0 : water droplet's diameter before impact

$D_{\max}$: size of the “puddle” formed at the maximal extension

Velocity U_0 just before impact : (Energy conservation)

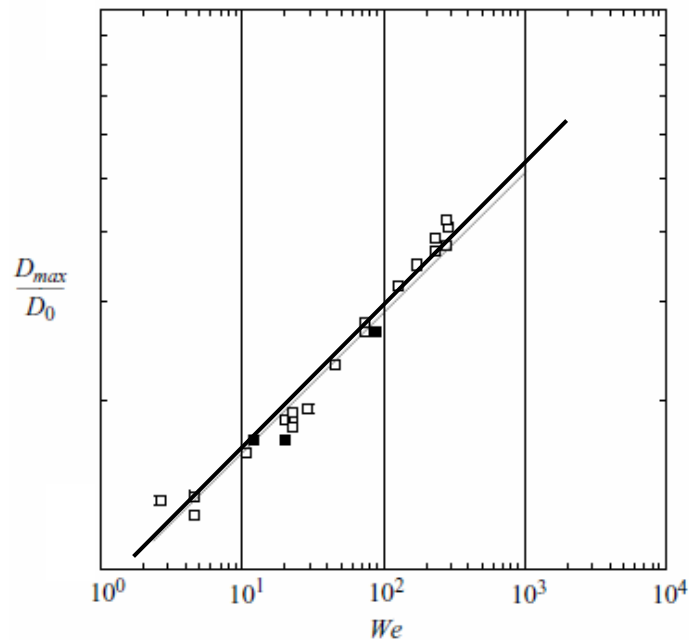
$$E_c = \frac{1}{2} \rho \frac{4}{3} \pi \left(\frac{D_0}{2} \right)^3 U_0^2 = E_{p,gravity} = \rho \frac{4}{3} \pi \left(\frac{D_0}{2} \right)^3 gh \quad \Rightarrow \quad U_0 = \sqrt{2gh}$$

Water droplet is assumed to be an harmonic oscillator : (pure transfer of kinetic energy into surface energy, we neglect the surface area of the initial drop)

$$E_c = \frac{1}{2} \rho \frac{4}{3} \pi \left(\frac{D_0}{2} \right)^3 U_0^2 = E_p = \frac{1}{2} \gamma D_{max}^2$$

$$D_{max} \approx \sqrt{W_e} D_0 = (W_e)^{1/2} D_0$$

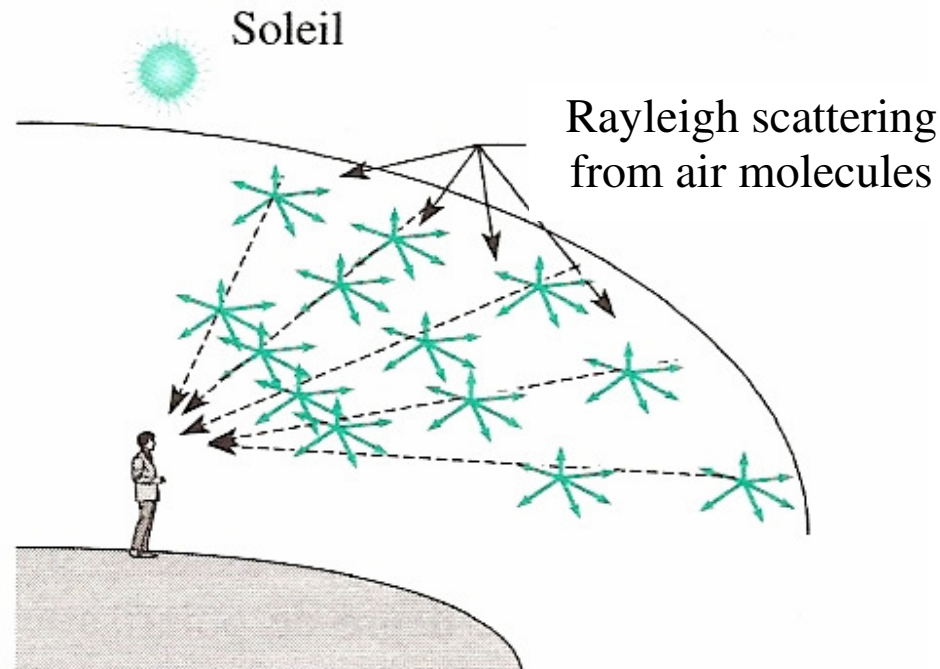
$$(W_e = \frac{\rho U_0^2 D_0}{\gamma}, \text{Weber number})$$



The solid line indicates the slope 1 / 2

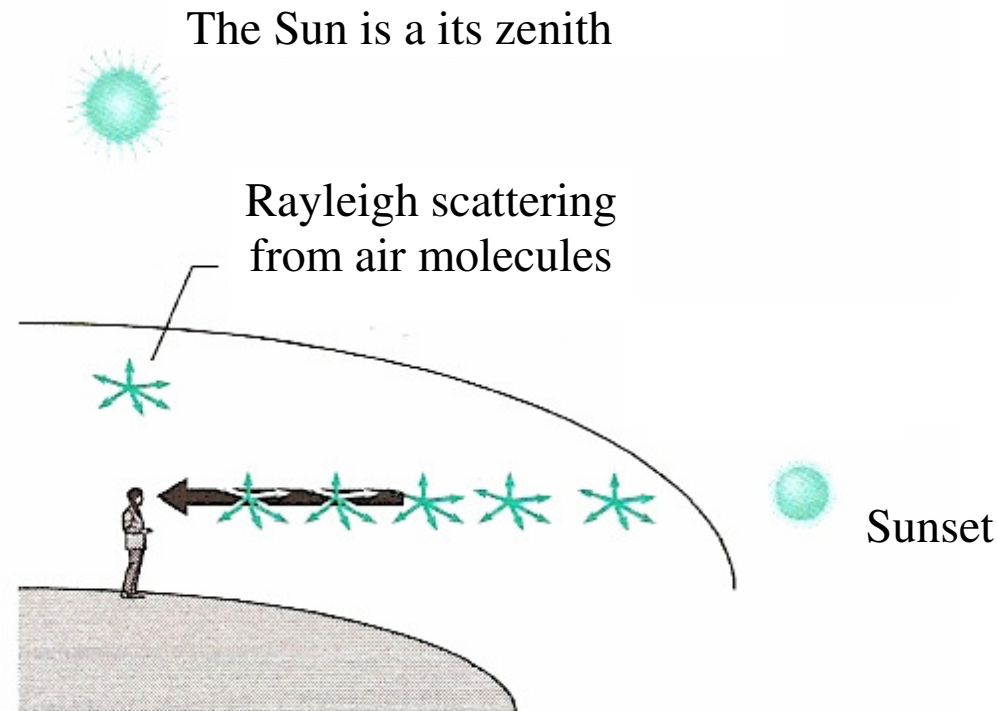
Why the sky is blue and the sunset red ? (Rayleigh diffusion, elastic electron model)

The strong wavelength dependence of Rayleigh scattering enhances the short wavelengths, giving us the blue sky.



$$P_{diff} = \frac{4\pi^3 \mu_0 c^3 p_0^2}{3} \frac{1}{\lambda^4}$$

Why the sky is blue ? $P_{diff} \approx \frac{1}{\lambda^4} : \lambda_{red} = 2\lambda_{blue} \Rightarrow P_{diff,blue} = 16P_{diff,red}$

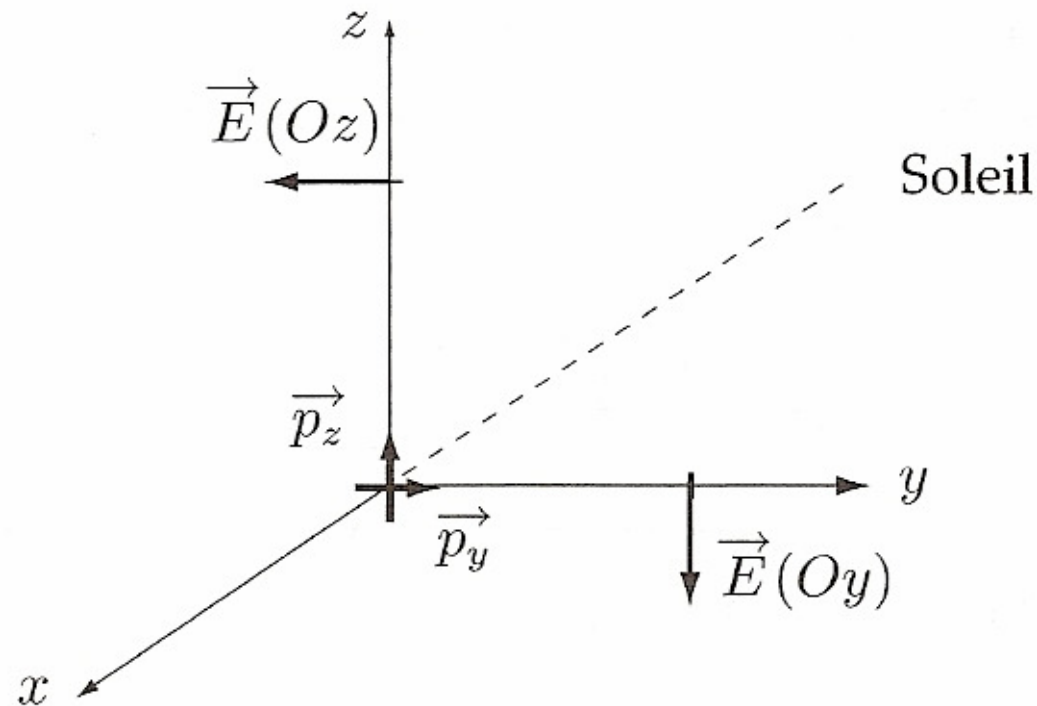


Sunsets are reddened because for sun positions which are very low or just below the horizon, the light passing at grazing incidence upon the earth must pass through a greater thickness of air than when it is overhead.

Since short wavelengths are more efficiently scattered by Rayleigh scattering, more of them are scattered out of the beam of sunlight before it reaches you : sunset is red !

Un dipôle rayonnant est une source de lumière anisotrope créant des ondes EM polarisées rectilignement dans le plan dipôle – observateur.

S'il est excité par de la lumière naturelle non polarisée rectilignement, on peut toujours considérer qu'il y a superposition de deux directions orthogonales quelconques.

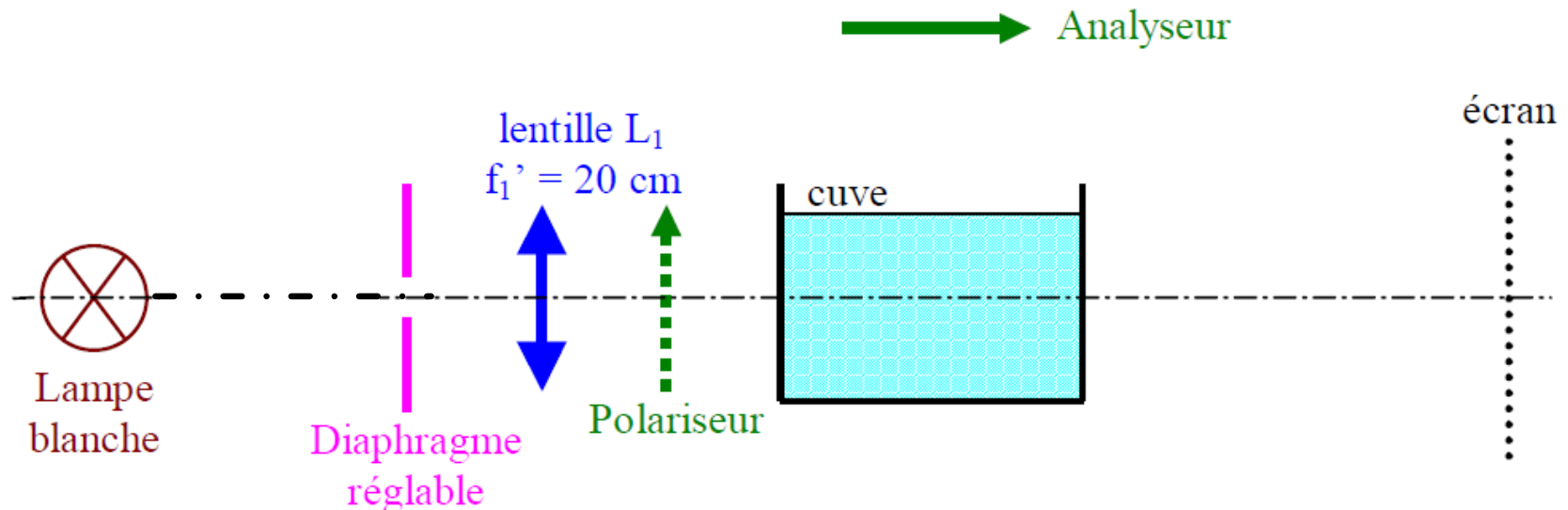


Setup experiment :

With no tank, get the geometrical image on the screen with the white light on.

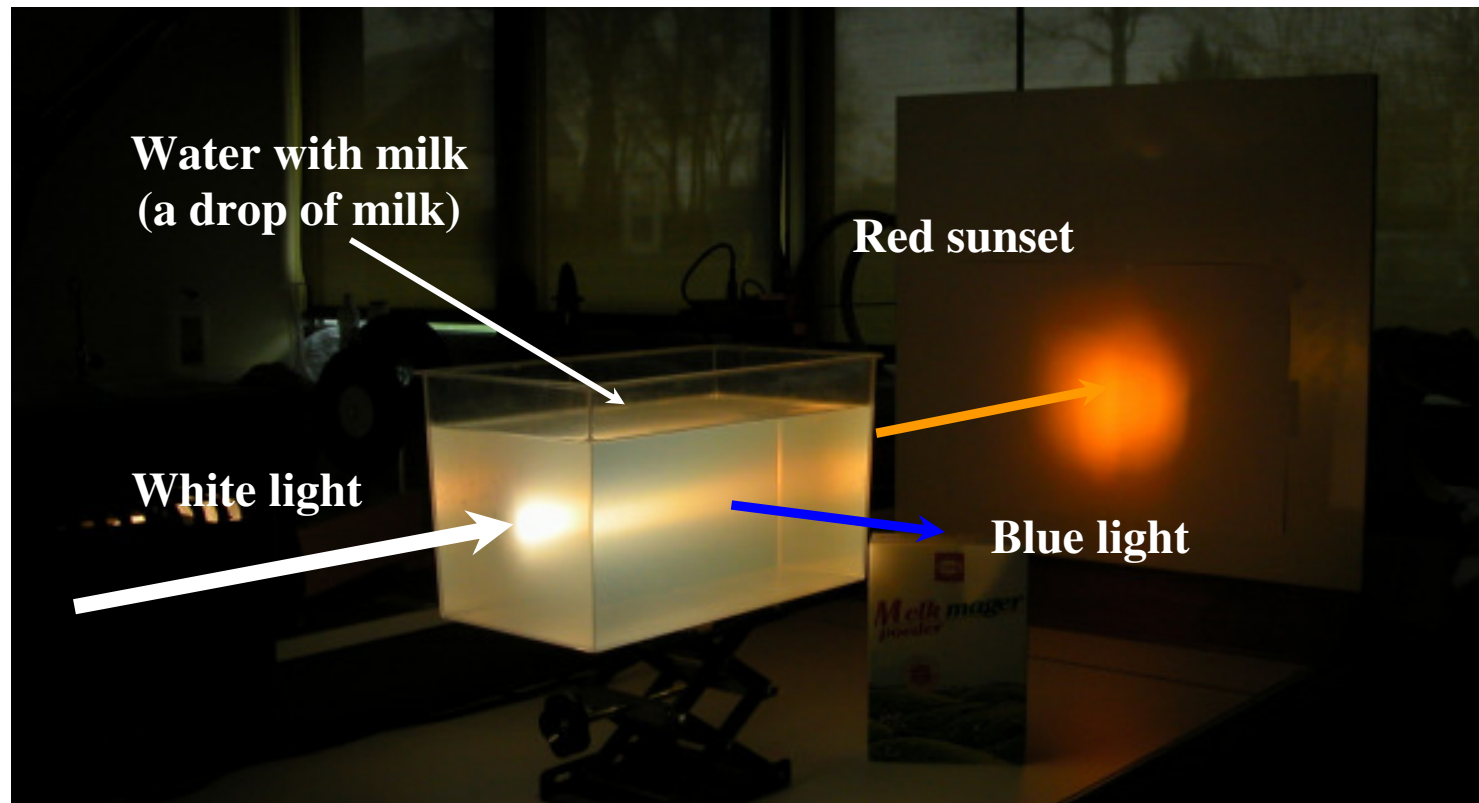
Put a tank full of water between the lens and the screen.

Drop a little milk inside



Put a polarizer before the tank to linearly polarize the light.

Look through an analyzer to analyse the light diffused perpendicularly to the propagation direction.



Video : « For the love of physics (40') »

Radial harmonic oscillator :

Some positive electric charges are set on an (Oxy) plane.

Using the field map from the Maple software, define their values denoted q_i and positions.

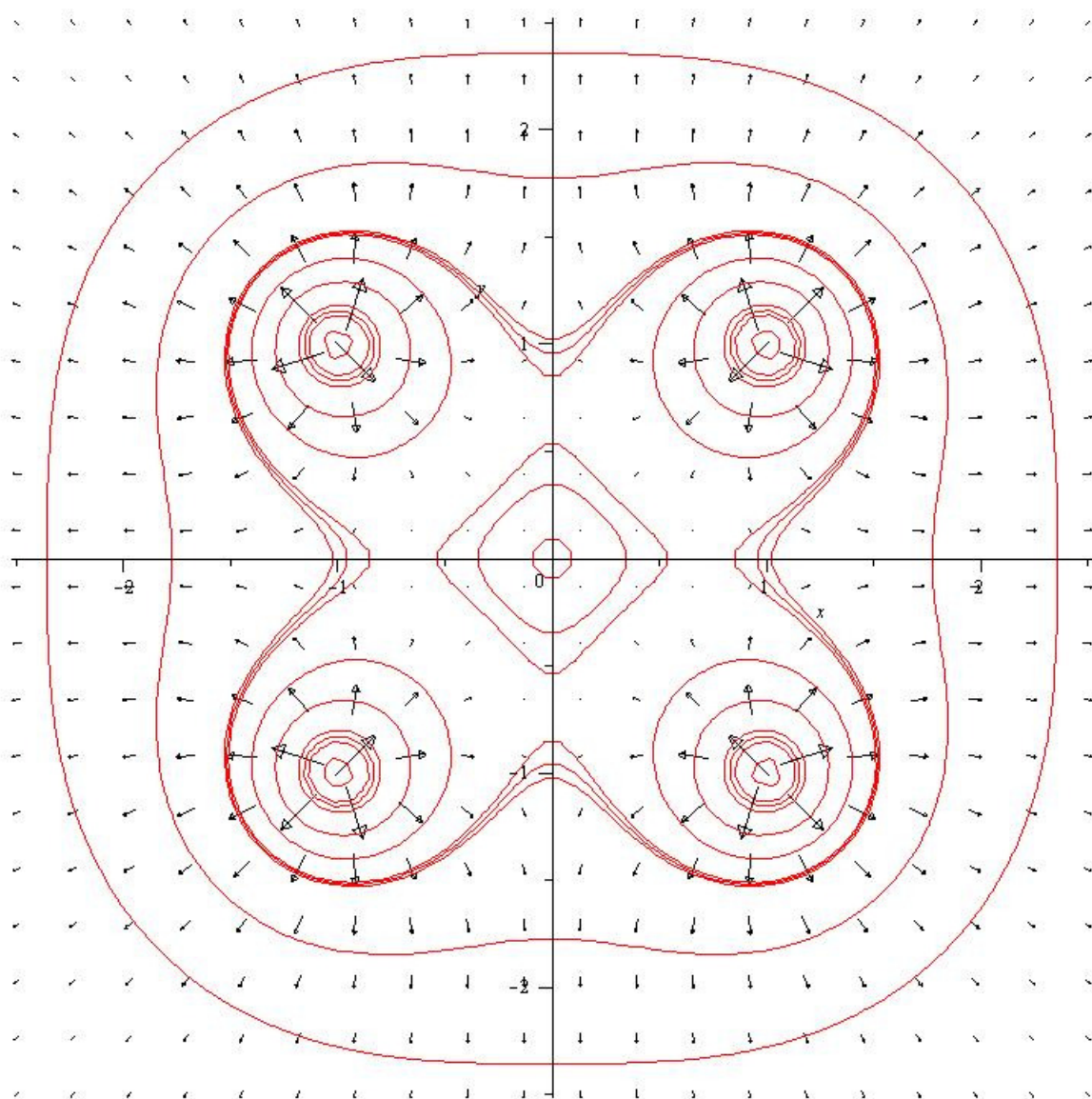
Comment on the equipotential lines.

Near the center of a square (of side a) to be defined, the electric potential can be written as :

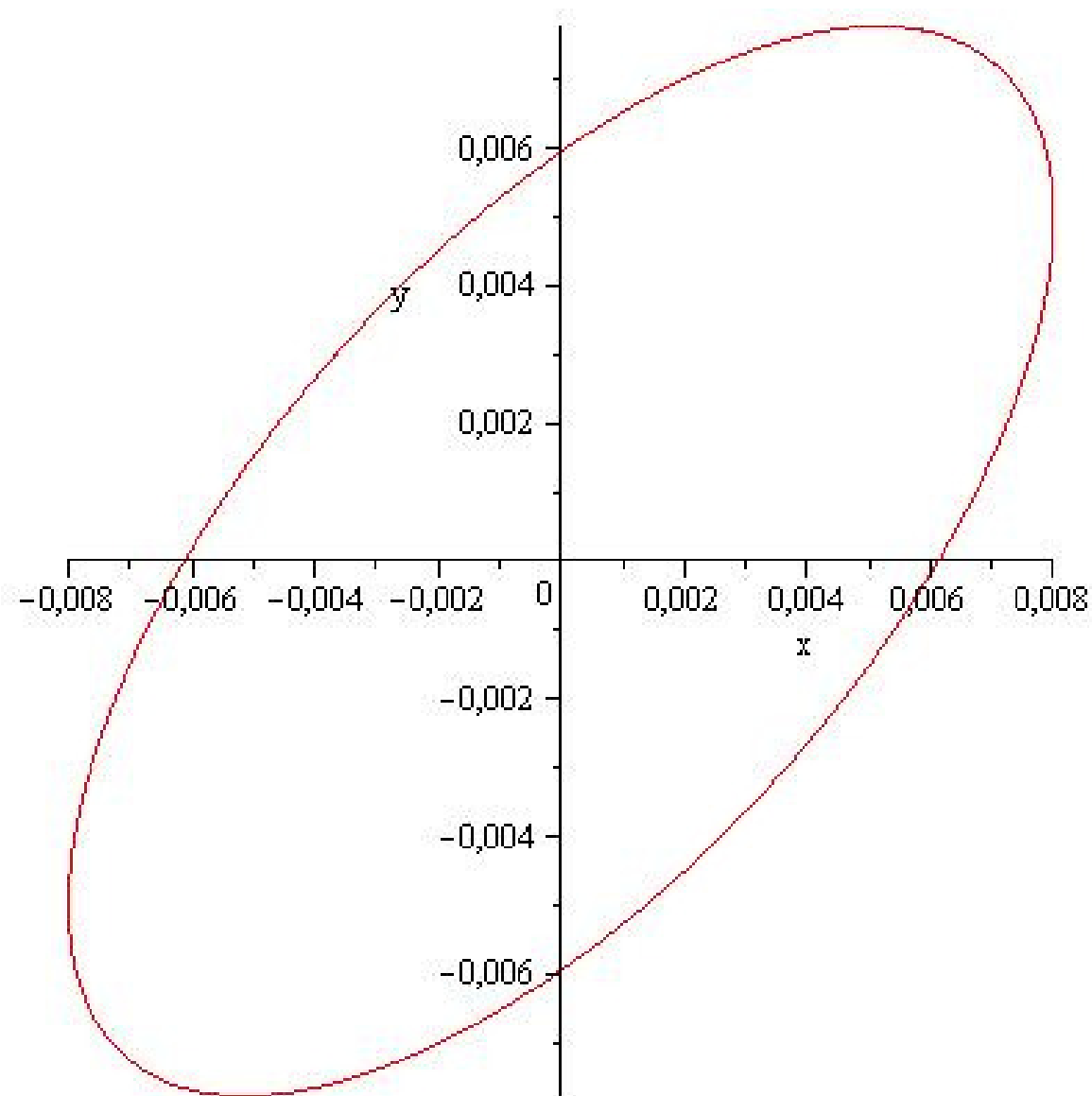
$$V(x, y, z) = V_0 + a_1 x + a_2 y + b_1 x^2 + b_2 y^2 + cxy$$

Using the symetry properties of the potential, define the values of the constants.

Study the motion of a positive charge set close to the square center.



Programme MAPLE



Programme MAPLE

To finish with a funny video : a MIT Video about a glass entering in resonance



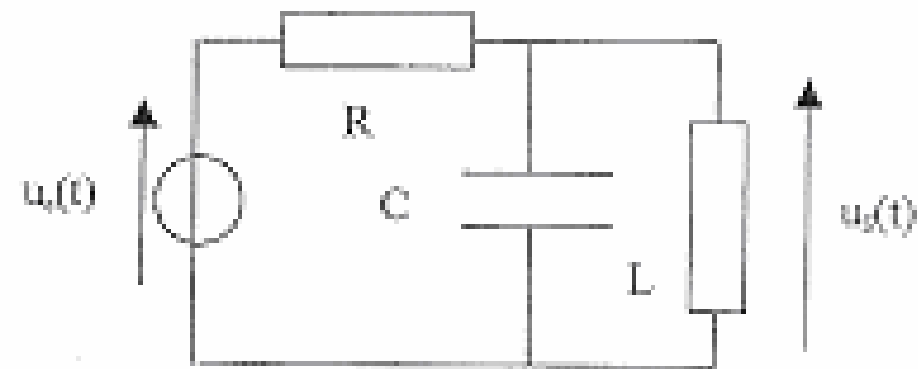
(Lecture 27, from 06:13 to 10:20)

TD Expérimentaux :

Etude expérimentale d'un circuit électrique.

On considère le circuit ci contre, constitué d'une capacité $C = 0,1\mu\text{F}$, d'une résistance R et d'une inductance L , en régime sinusoïdal forcé.

On observe à l'oscilloscope la tension d'entrée $u_e(t) = E \cos(\omega t)$ sur la voie 1 et la tension de sortie $u_s(t)$ sur la voie 2. On dispose en outre d'un multimètre numérique.



1) On mesure la tension d'entrée au multimètre numérique en position AC. Celui-ci indique 3,535 V.

Quelle est l'amplitude de la voie 1 en carreaux sachant que son calibre est de 2V/division ?

2) Déterminer l'expression de $u_s(t)$

3) On se place en mode X-Y et on observe un segment de droite sur l'oscilloscope pour une fréquence de 1592 Hz

3-a) Qu'observerait-on à une fréquence quelconque ?

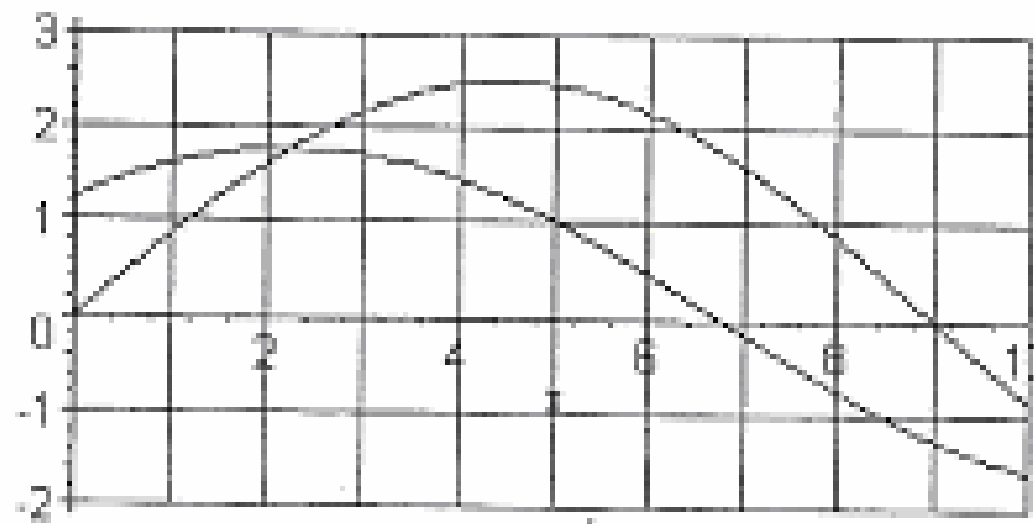
3-b) Dédurre de la mesure la valeur de L .

4) On se place à la fréquence de 796 Hz et on visualise sur l'oscilloscope la figure ci-dessous (les calibres sur les deux voies sont les mêmes).

4-a) Identifier les courbes. Quelle tension est en avance ?

4-b) Déterminer le déphasage (en degrés) entre la tension de sortie et la tension d'entrée. Quelle méthode utilisez vous ? (le premier zéro de la courbe de plus petite amplitude se situe à 6,75 carreaux)

4-c) En déduire la valeur de la résistance R.



5) On débranche la voie 2 et la courbe restante défile. Que se passe-t-il ? Sur quelle touche de l'oscilloscope faut il appuyer pour observer une courbe stable?

6) Sur le générateur on constate que la touche offset (décalage en tension) est activée. Cela s'observe-t-il sur la figure ci dessus ? Quelle touche de l'oscilloscope faut il activer pour visualiser ce décalage ?

La tension de sortie est elle décalée en tension ?

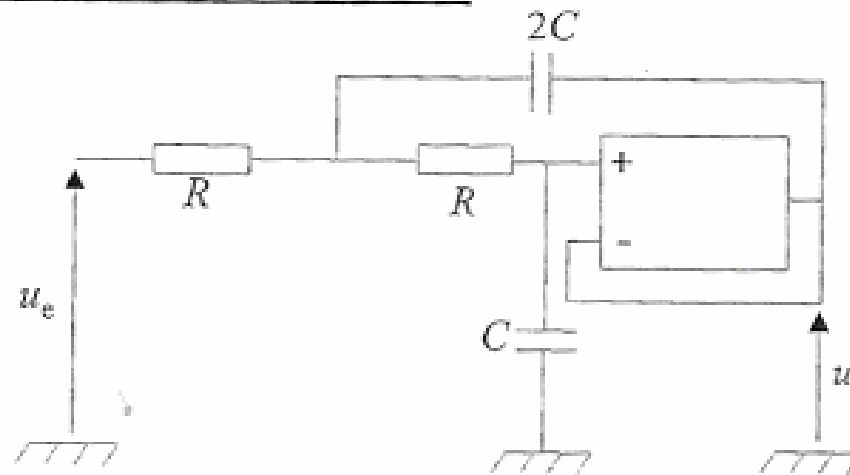
7) La tension d'entrée est un échelon de tension $u_e(t) = E H(t)$. Déterminer le caractère pseudo périodique ou apériodique de la tension de sortie.

Étude d'un filtre de Butterworth

On considère le montage ci-contre dans lequel l'A.O est supposé être en fonctionnement linéaire.

a) On pose $x = \sqrt{2}RC\omega$. Montrer que sa fonction de transfert s'écrit

$$H(jx) = \frac{u_s}{u_e} = \frac{1}{1 - x^2 + jx\sqrt{2}}$$



b) Tracer rapidement les diagrammes de Bode du gain et de l'avance de phase φ de u_s sur u_e (étude asymptotique), en fonction de $\log(x)$. De quel type de filtre s'agit-il ?
Déterminer la fréquence de coupure f_c .

A.N : pour $R = 10 \text{ k}\Omega$ et $C = 7,5 \text{ nF}$ on a $f_c = 1500 \text{ Hz}$

c) On envoie en entrée du montage un signal triangulaire d'amplitude E et de fréquence f_0 .
On donne son développement en série de Fourier :

$$u_e(t) = \frac{8E}{\pi^2} \left[\sin(2\pi f_0 t) - \frac{1}{9} \sin(2\pi(3f_0)t) + \frac{1}{25} \sin(2\pi(5f_0)t) - \frac{1}{49} \sin(2\pi(7f_0)t) + \dots \right]$$

Quel est le bon oscillogramme parmi les quatre proposés ? Justifier

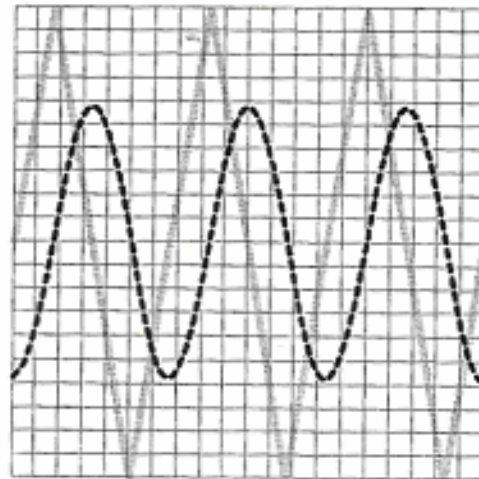


figure i)

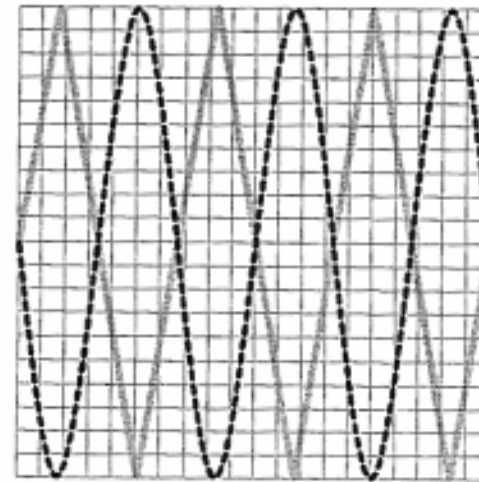


figure ii)

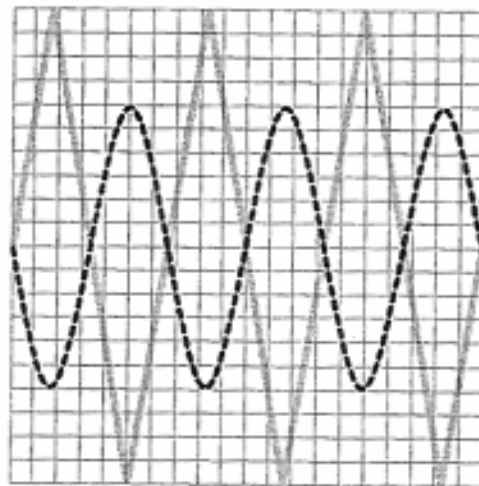


figure iii)

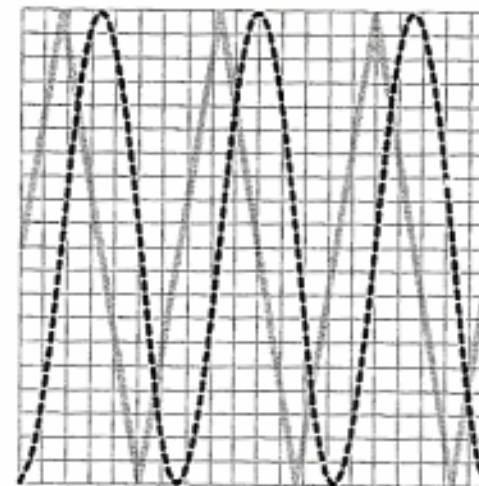


figure iv)

On a la même sensibilité verticale pour u_s et u_e , et une base de temps de $100 \mu s$ par carreau.
Les zéros sont réglés au centre de l'écran.