

# **« Autour de l'optique géométrique et ondulatoire »**

**« About geometrical and physical optics »**

**Cours de méthodologie – Ecole d'été (ITC Phnom Penh)**

**Olivier GRANIER**

**(Du lundi 20 au mercredi 29 août 2012)**

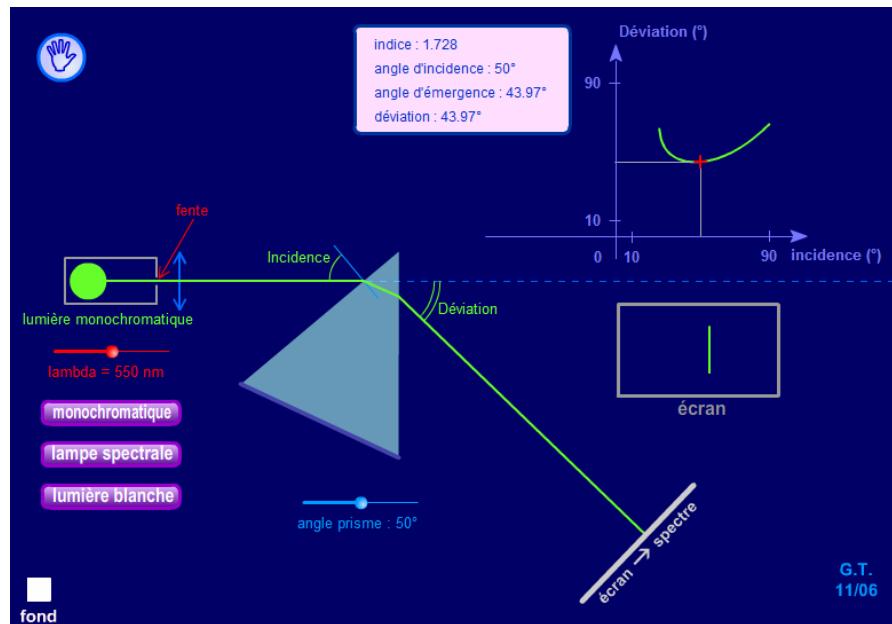
## Vidéo : « Le goniomètre à prisme »

Animation JJR :

### Animations JJR/Optique géométrique/Prismes/Prisme dispersif

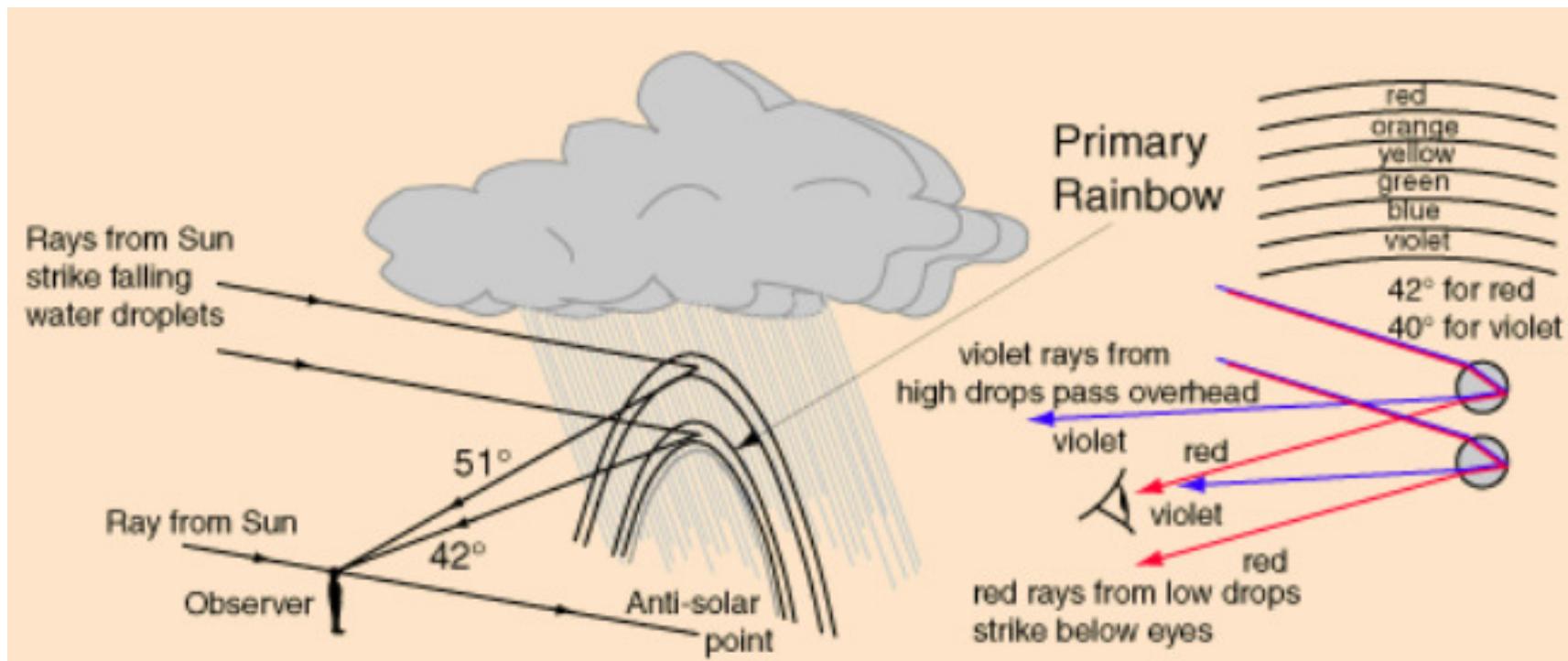
Animation Geneviève Tulloue :

### Déviation par un prisme (deviation by a prism)



- Light changes speed as it moves from one medium to another
- The refractive index varies with the wavelength of the light used (dispersion phenomenon, creating an effect similar to a rainbow)
- This can be used to separate a beam of white light into its constituent spectrum of colors

## Primary Rainbow

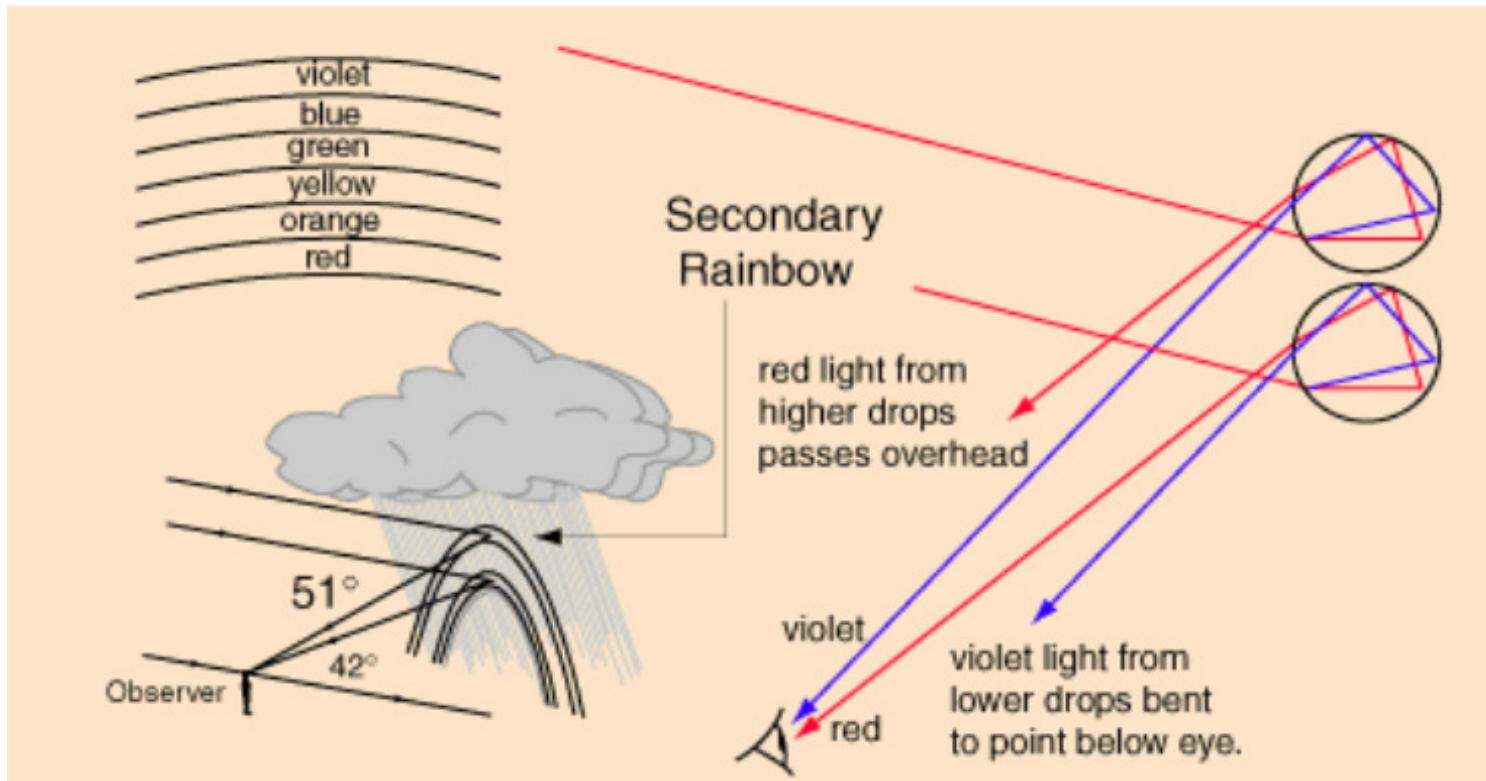


The primary rainbow forms between about  $40^\circ$  and  $42^\circ$  from the antisolar point.

The light path involves refraction and a single reflection inside the water droplet.

We can observe this primary rainbow as colored arches opposite to the sun.

## Secondary Rainbow

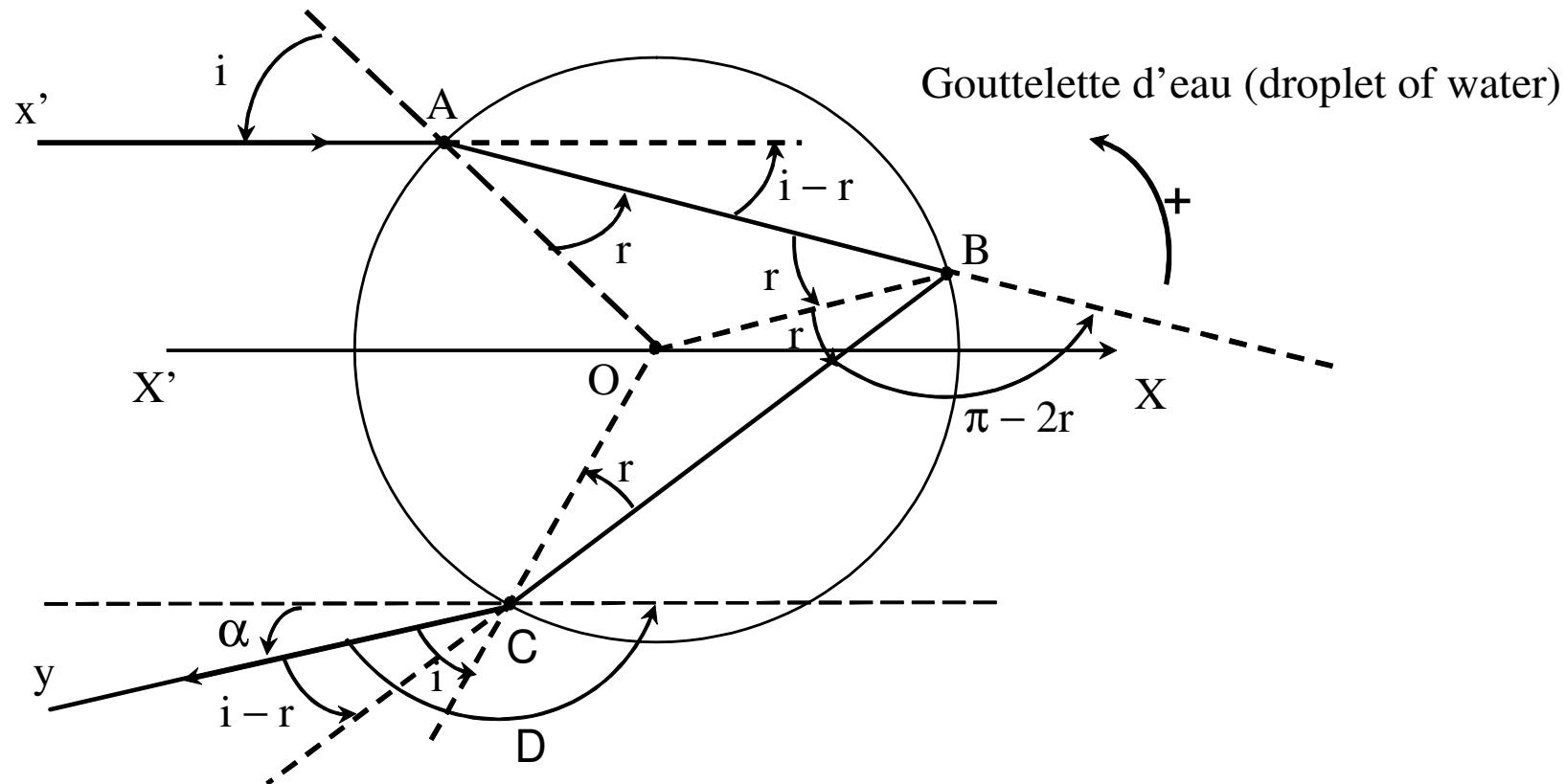


The secondary rainbow is about  $10^\circ$  further out from the antisolar point than the primary rainbow. It is about twice as wide and has its colors reversed.

There are two reflections inside the water droplet.

The light of the secondary rainbow is one-tenth the intensity of that the primary rainbow.





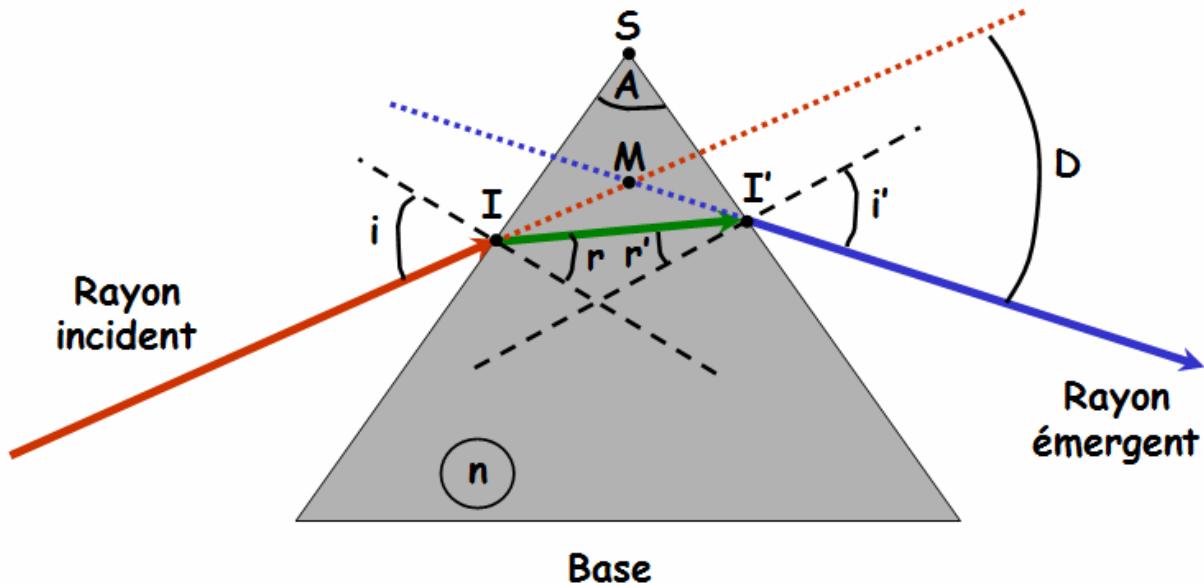
When the ray goes outside the droplet, the value of the angular deviation D is :

$$D = 2(i - r) + (\pi - 2r) = \pi + 2i - 4r$$

The angle  $\alpha$  defined on the figure is :

$$\alpha = \pi - D = 4r - 2i$$

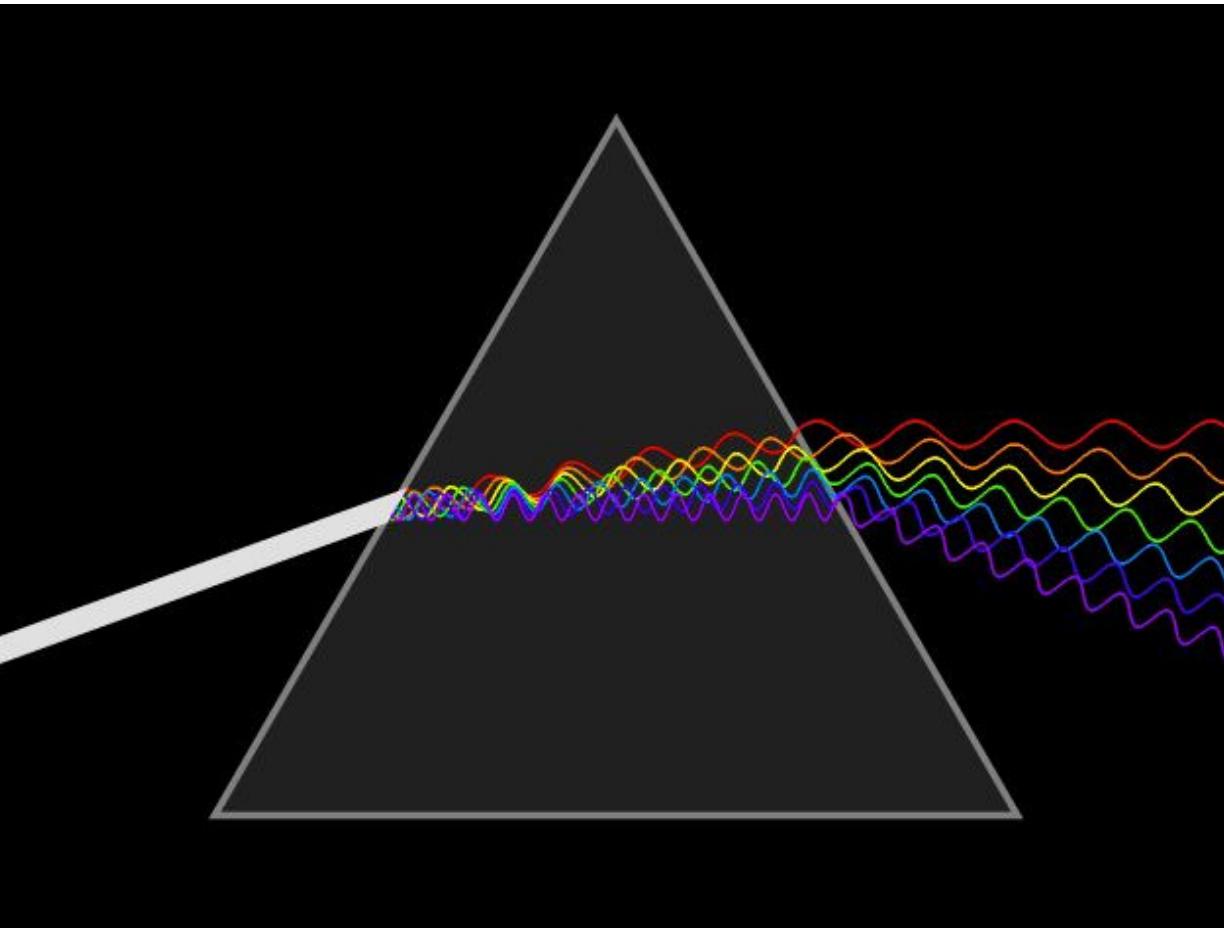
## Snell-Descartes's law and deviation angle :



$$\sin i = n \sin r \quad \sin i' = n \sin r' \quad D = i + i' - A \quad r + r' = A$$

$$\frac{\pi}{2} \geq i \geq i_0 \quad \text{avec} \quad \sin i_0 = n \sin(A - i_\ell)$$

$$n = \frac{\sin\left(\frac{D_m + A}{2}\right)}{\sin\frac{A}{2}}$$

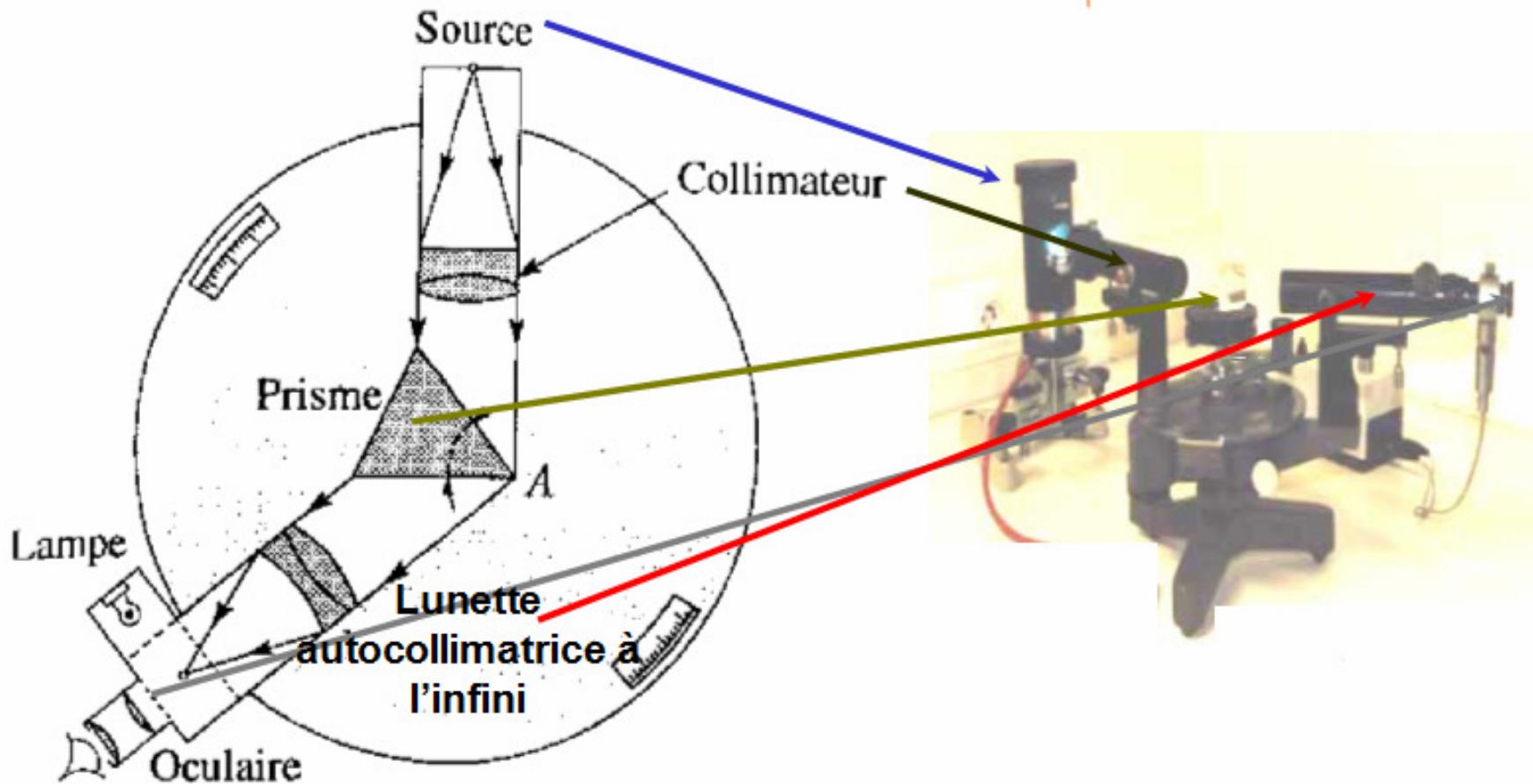


*A triangular prism, dispersing light.*

*Waves shown to illustrate the different wavelengths of light.*

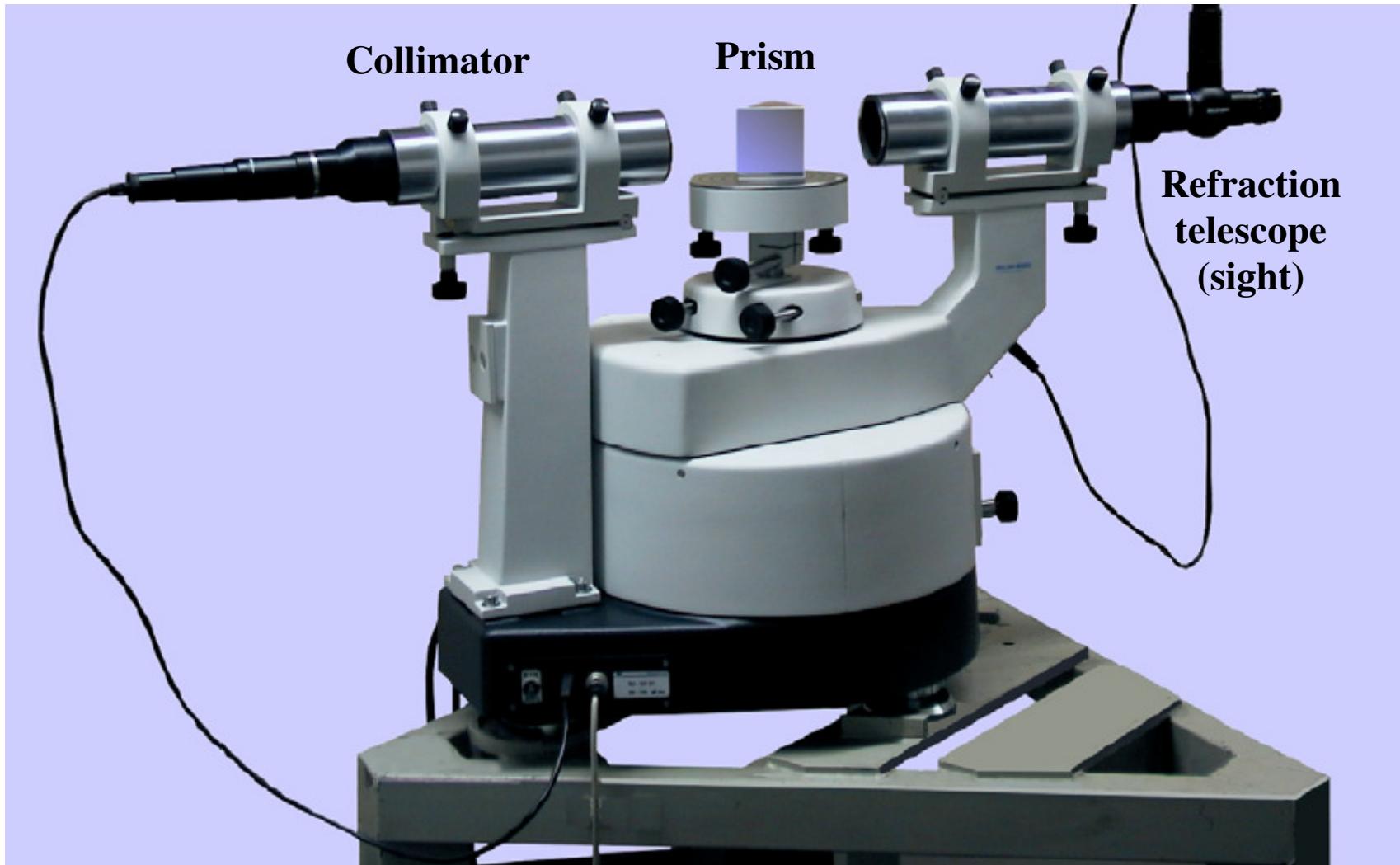
*Longer wavelengths (red) are refracted less than shorter wavelengths (violet)*

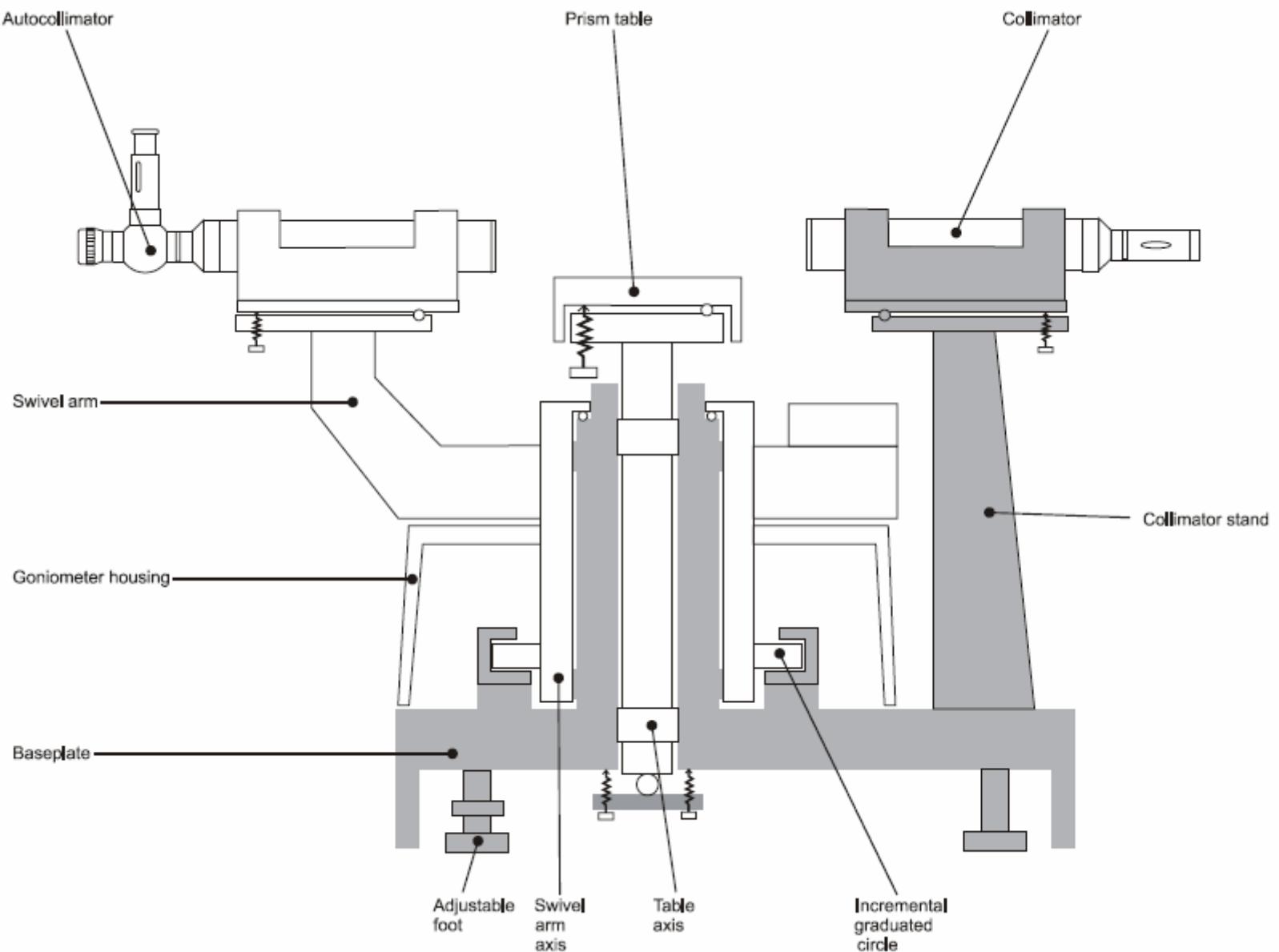
## Prism goniometer :



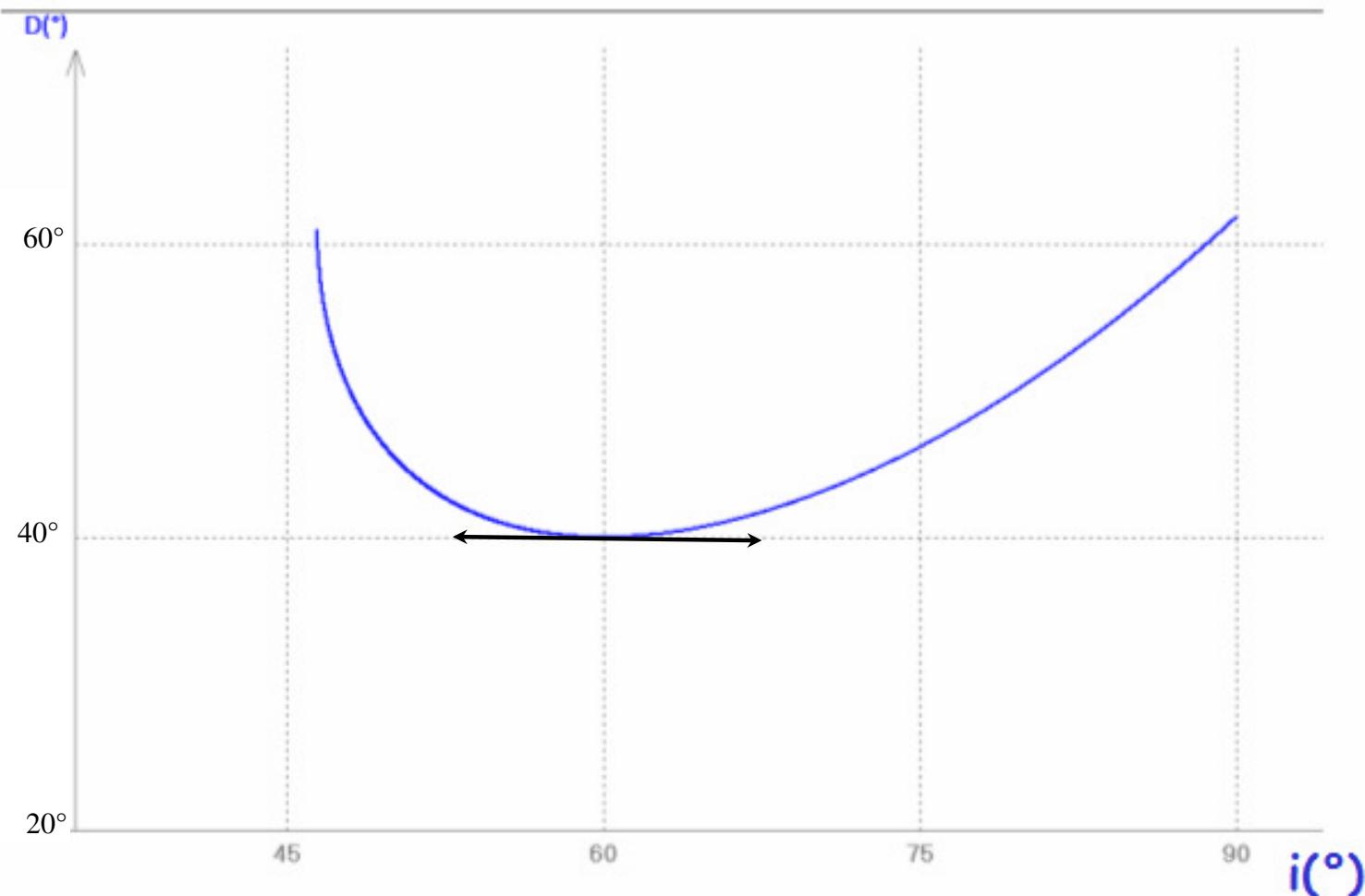
*A goniometer is an instrument that measures angles and refractive index.*

*(The term goniometer is derived from two Greek words, *gōnia*, meaning angle, and *metron*, meaning measure)*



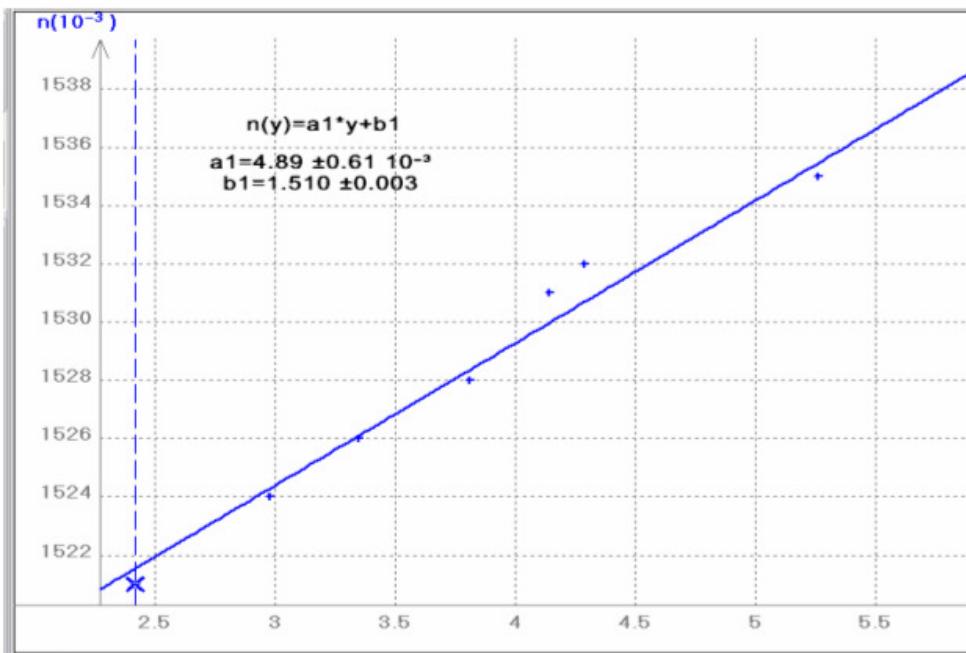


## Deviation curve ( $D = D(i)$ , for a given wavelength) :



$\lambda$ (nm)	404,7	435,9	483	491,6	512	546,1	579	643
$D_m$	40,61	40,26	40,00	39,90	39,64	39,46	39,28	39,01
$n$	1,539	1,535	1,532	1,531	1,528	1,526	1,524	1,521
$1/\lambda^2$ (10 <sup>-6</sup> nm <sup>-2</sup> )	6,11	5,26	4,29	4,14	3,81	3,35	2,98	2,42

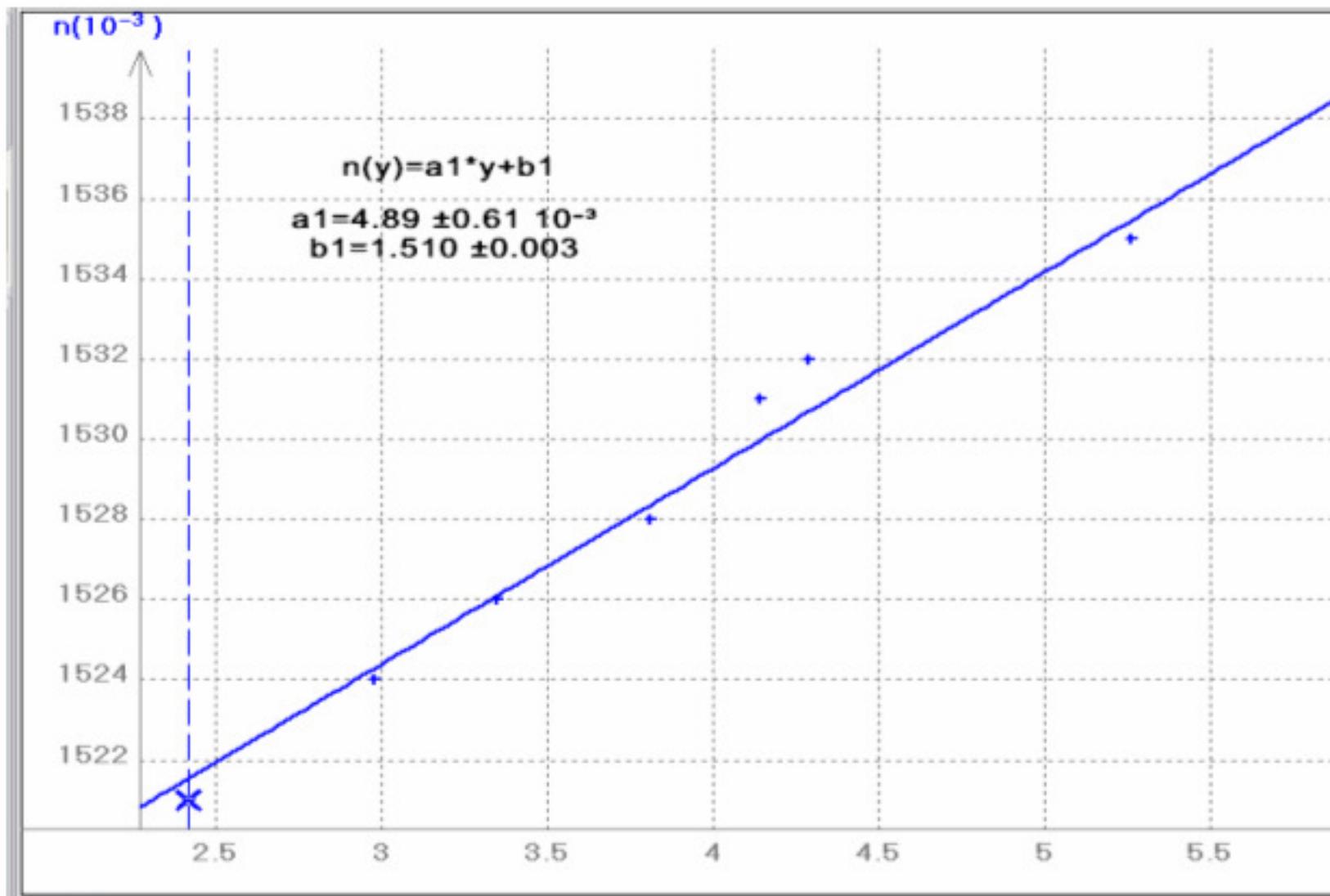
## Loi de Cauchy (Cauchy's law) – (With A = 60°)



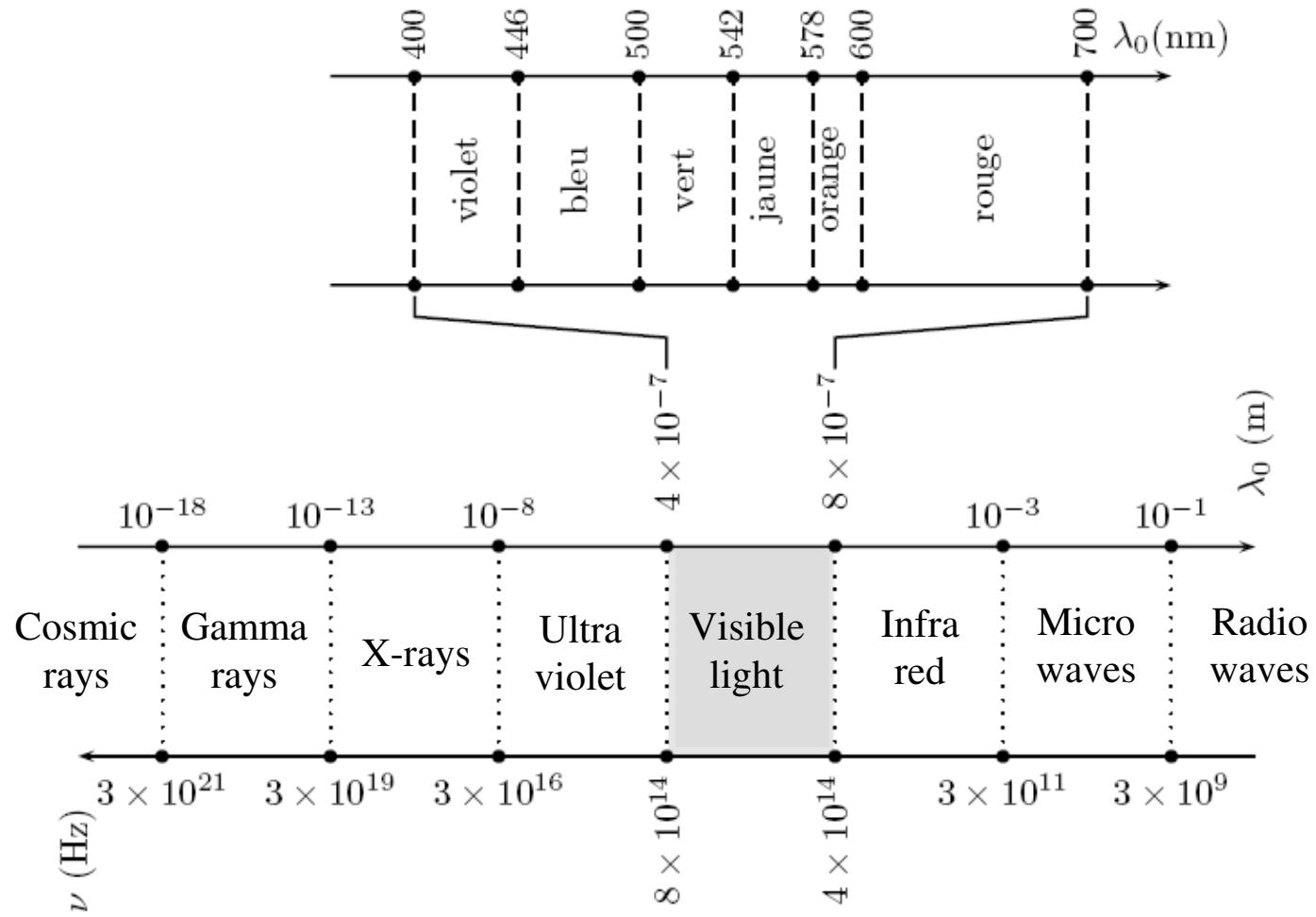
$$n = \frac{\sin\left(\frac{D_m + A}{2}\right)}{\sin\frac{A}{2}}$$

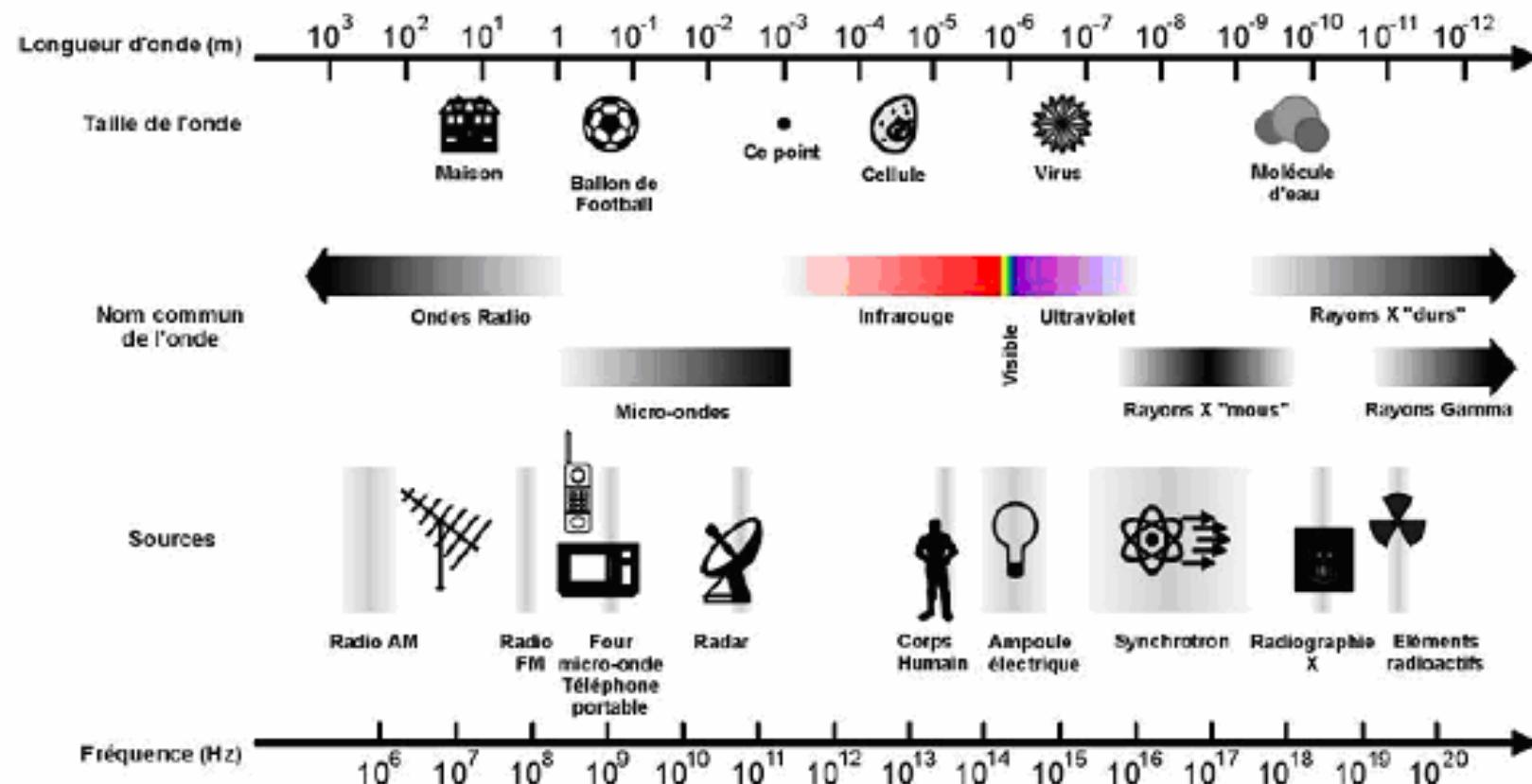
$$n(\lambda) = n_0 + \frac{B}{\lambda^2}$$

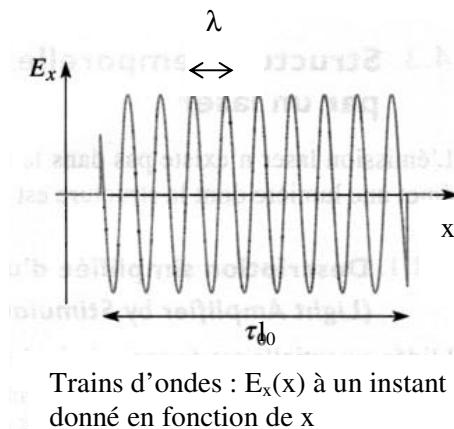
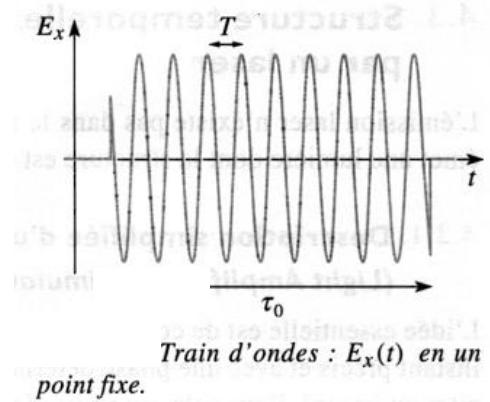
$$\Rightarrow \frac{\Delta n}{n} ?$$



## Coherent light :







$\tau_c$  : coherence time

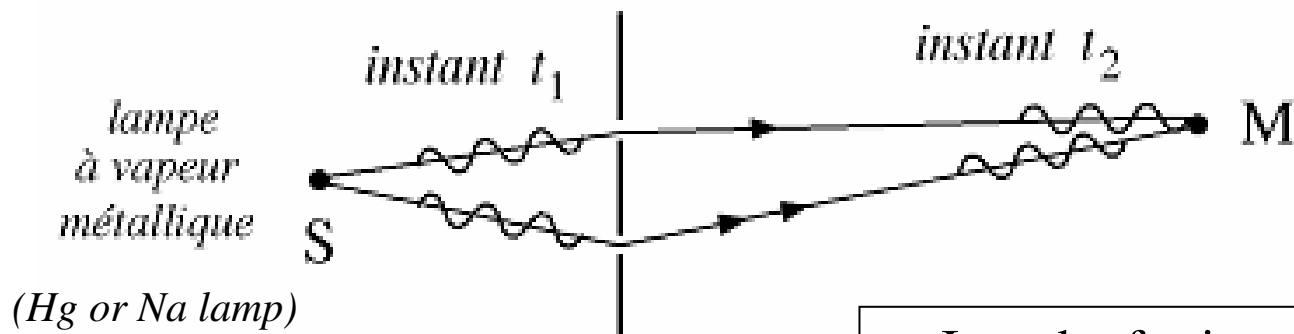
$L_c$  : coherence length

	$\lambda_0$ (nm)	radiation	$\nu_0$ (en $10^{14}$ Hz)	$\Delta\nu_0$ (en Hz)	$\tau_c$ (s)	$L_c$	$L_c$
Lumière blanche	400 à 800		7,5 – 3,5	$4.10^{14}$	$2,5.10^{-15}$	750 nm	$\approx \lambda_0$
Vapeur de mercure (haute pression)	546,1	Verte	5,49	$10^{12}$	$10^{-12}$	0,3 mm	$\approx 550\lambda_0$
Vapeur de mercure (basse pression)	546,1	Verte	5,49	$10^9$	$10^{-9}$	0,3 m	$\approx 5.10^5 \lambda_0$
Vapeur de cadmium	643,8	Rouge	4,66	$10^9$	$10^{-9}$	0,3 m	$\approx 5.10^5 \lambda_0$
Laser He – Ne ordinaire	632,8	Rouge	4,74	$10^9$	$10^{-9}$	0,3 m	$\approx 5.10^5 \lambda_0$
Laser He – Ne stabilisé	632,8	Rouge	4,74	$10^4$	$10^{-4}$	30 km	$\approx 5.10^{10} \lambda_0$

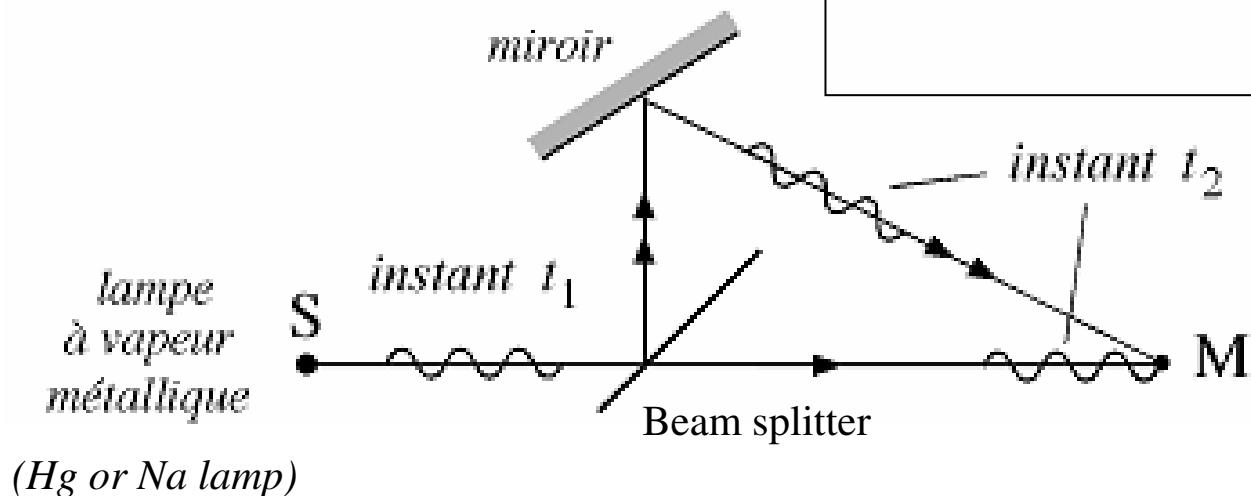
Relationship between coherence time and bandwidth :

$$\tau_c \Delta\nu_0 \approx 1 \quad ; \quad L_c = c\tau_c = \frac{\lambda_0^2}{\Delta\lambda_0}$$

*The two waves are coherent :*



*The two waves are incoherent :*



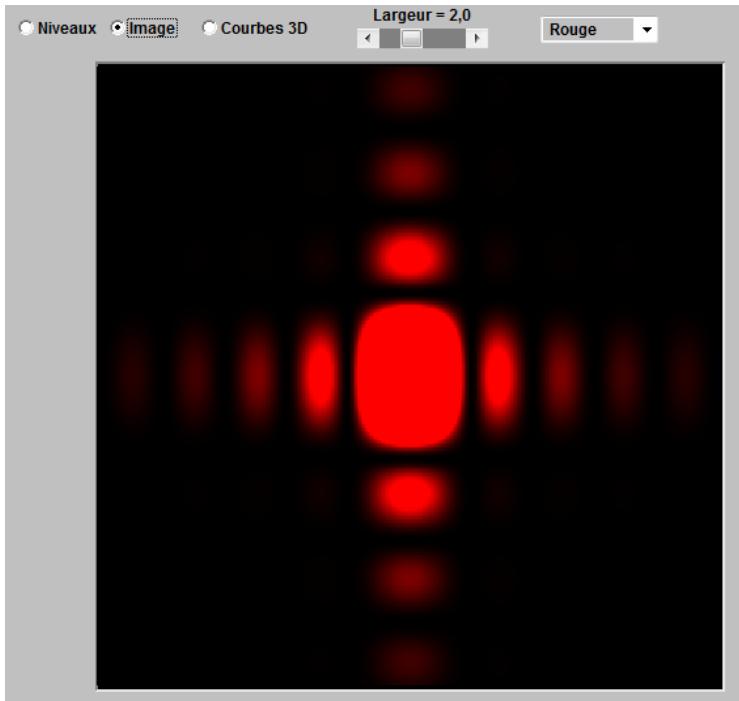
In order for interference to occur, the path length difference must be smaller than the coherence length :

$$\delta < L_c$$

**LASER Beam is a coherent and monochromatic source**

## Vidéo : « Diffraktion de Fraunhofer »

The phenomenon that the propagation of light deviates from the predictions by geometrical optics in the case of encountering an obstacle, either transparent or opaque, is called diffraction of light.



*No-one has ever been able to define the difference between interference and diffraction satisfactorily.*

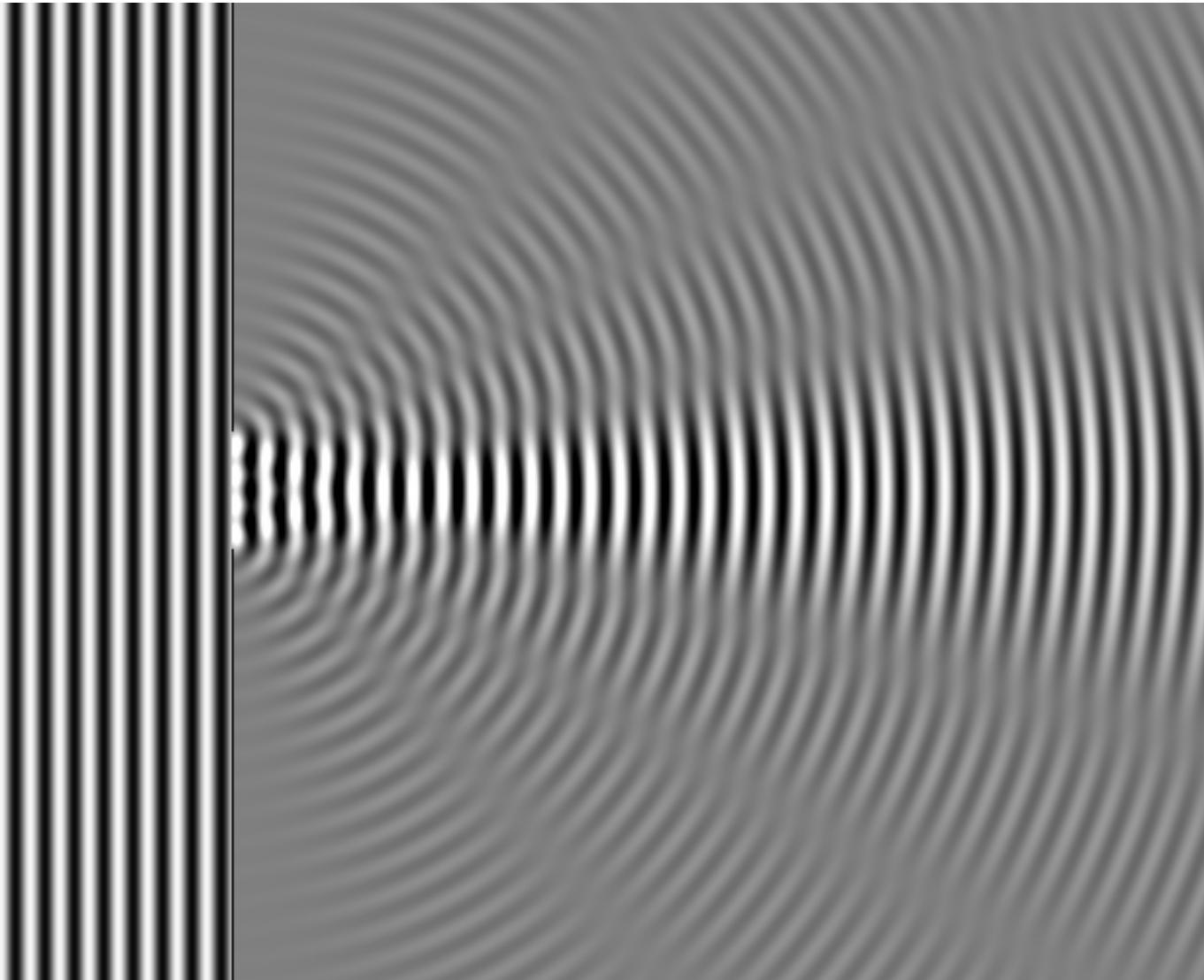
*It is just a question of usage, and there is no specific, important physical difference between them.*

(Richard Feynman, 1963)

Diffraction pattern

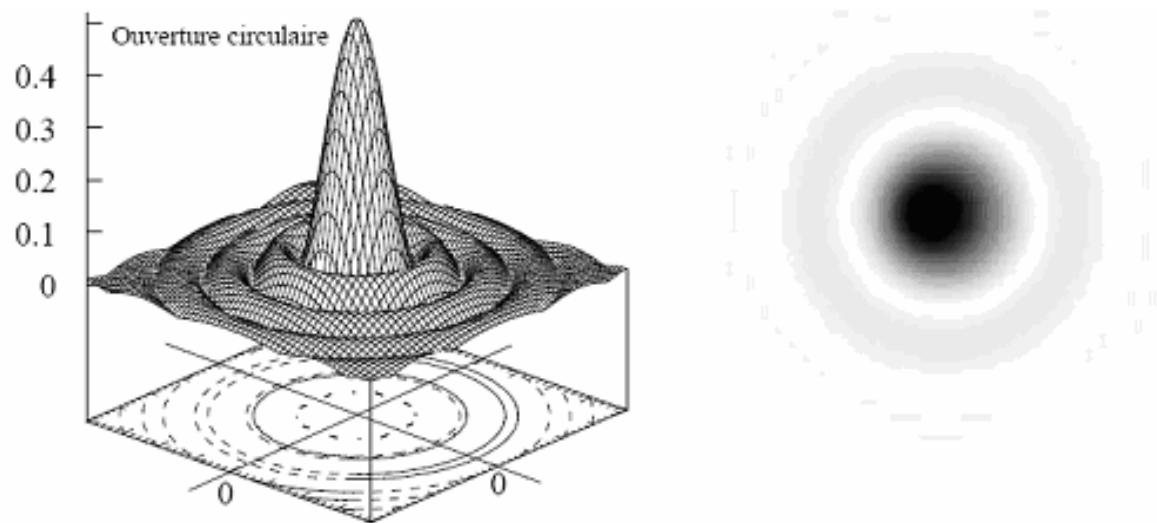
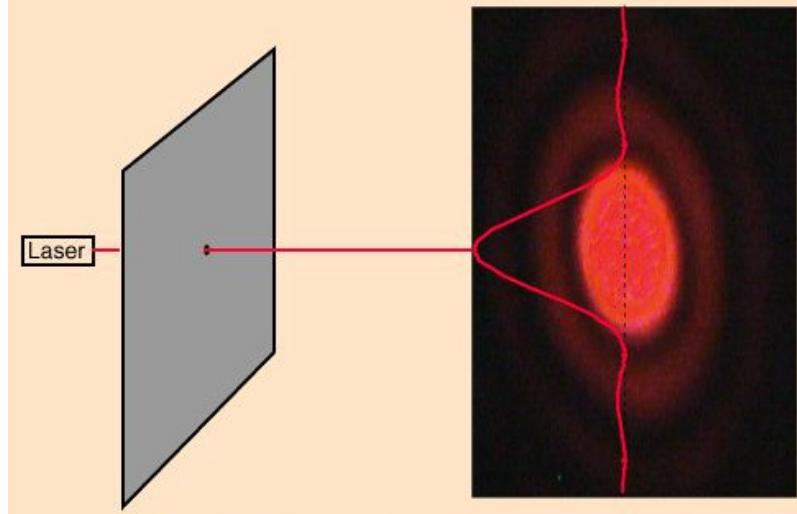
Animation JJR :

[Animations JJR/Optique ondulatoire/Diffraction/Diffraction par une pupille rectangulaire  
\(rectangular aperture\)](#)



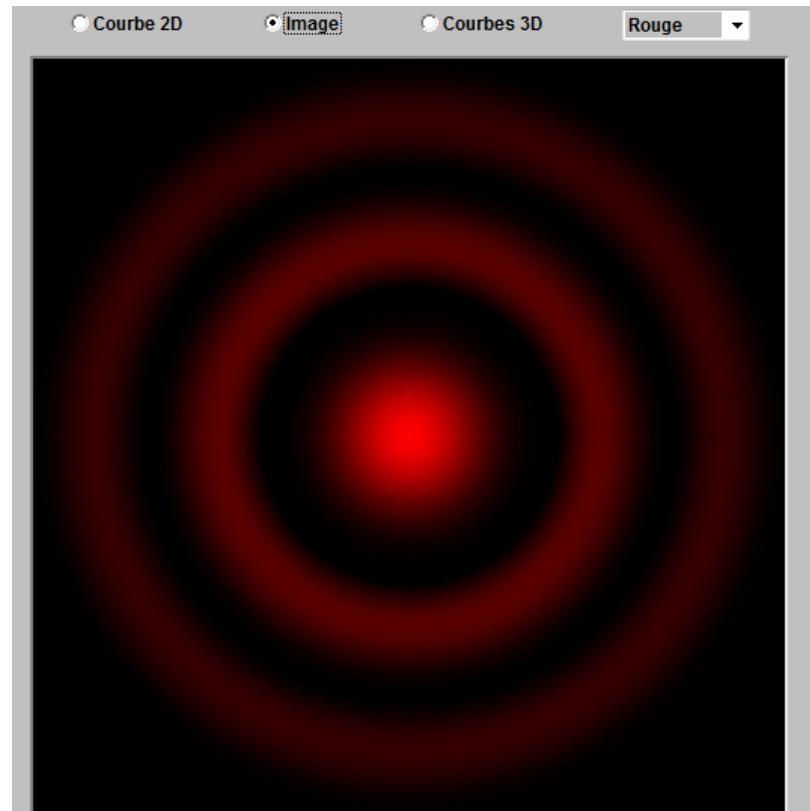
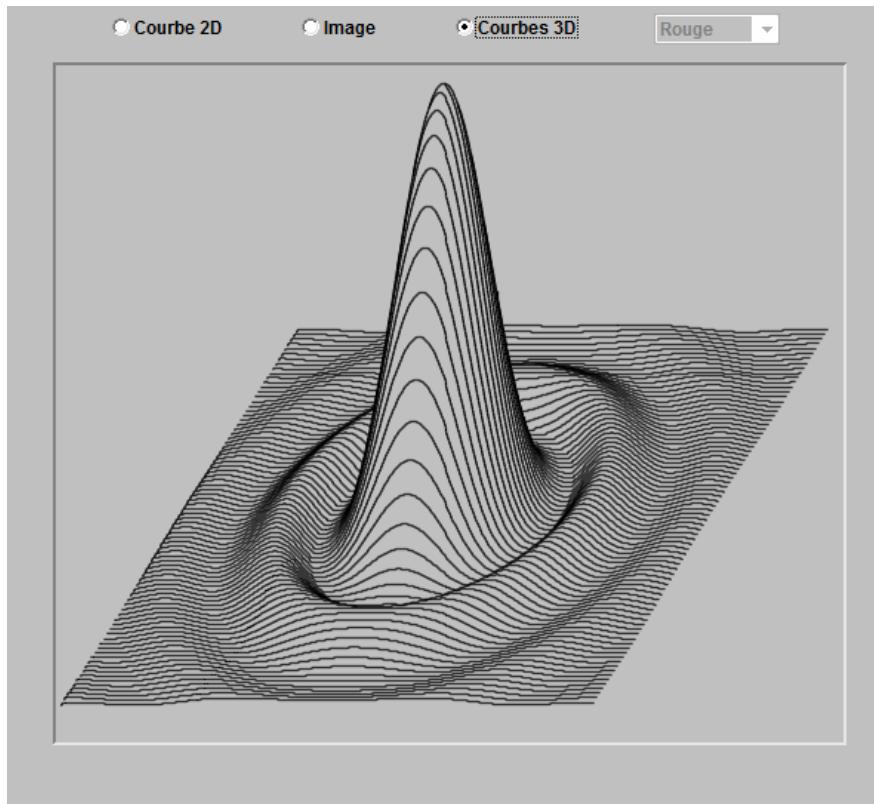
*Computational model of an interference pattern from two-slit diffraction*

## Circular Aperture Diffraction



## Animation JJR :

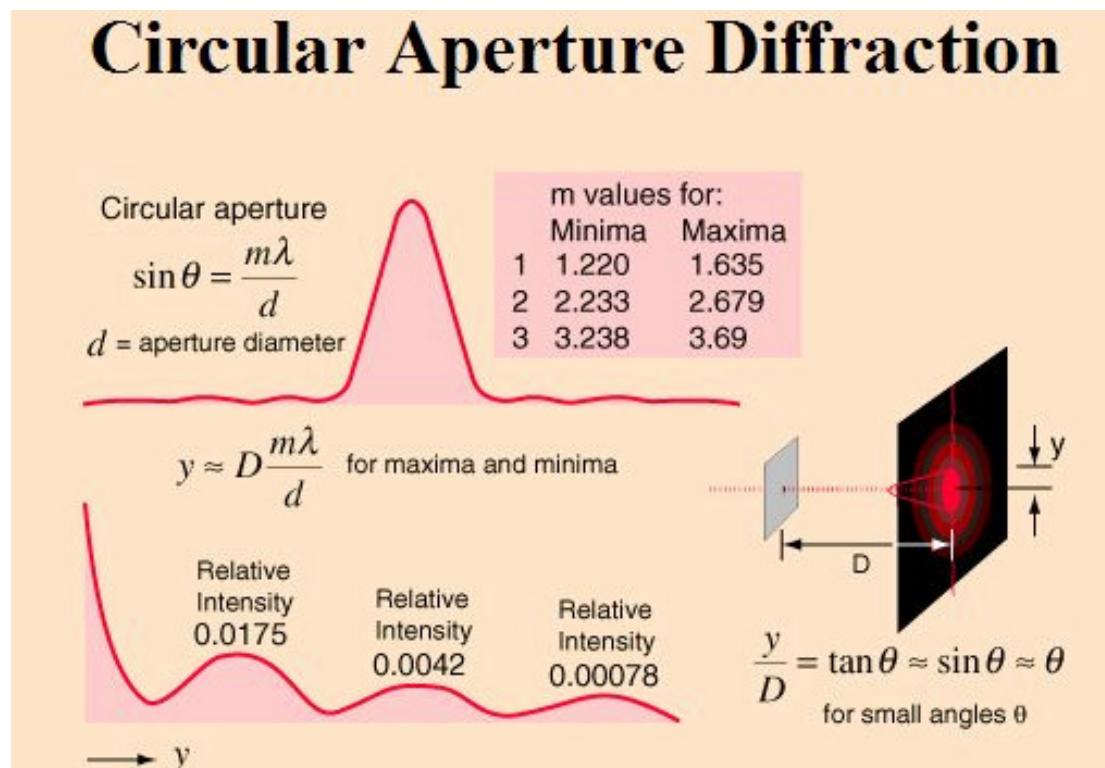
### Animations JJR/Optique ondulatoire/Diffraction/Diffraction par une pupille circulaire (Circular aperture)



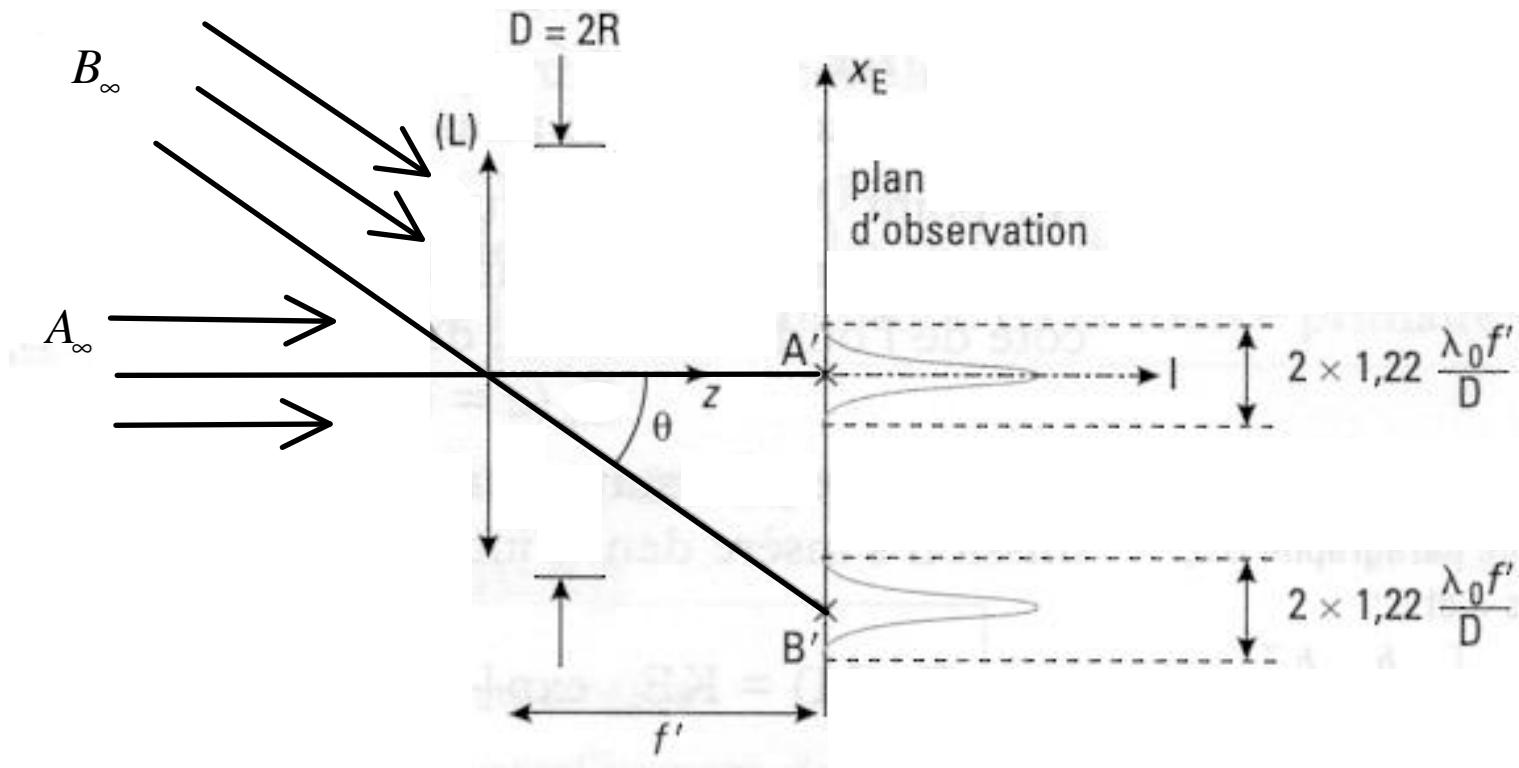
When light from a point source passes through a small circular aperture (pin hole) :

- Diffuse circular disc (Airy's disc) surrounded by much fainter concentric circular rings.
- Angular resolution : (D, diameter of the circular aperture)

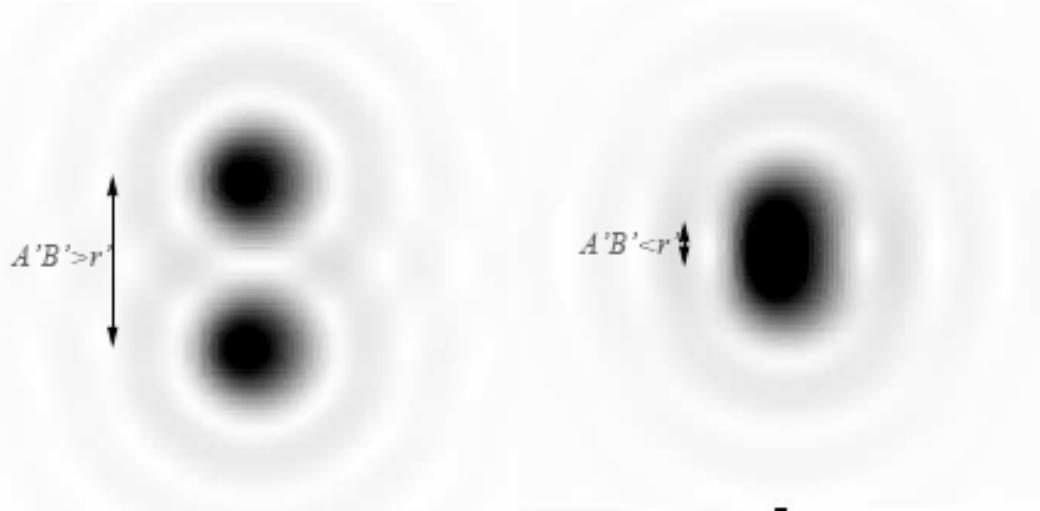
$$\sin \theta_L = 1,22 \frac{\lambda_0}{D}$$



- Great importance because the eye and many optical instruments have circular apertures.
- Limiting resolution of a system can be calculated using the Rayleigh criterion.



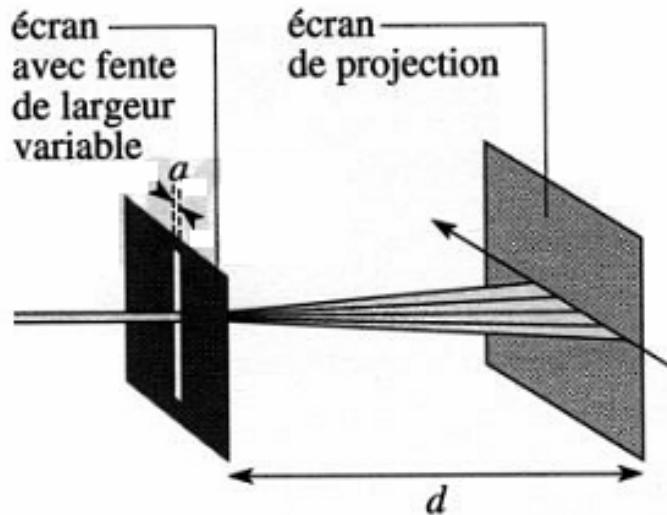
$$\theta > \theta_0 = 1,22 \frac{\lambda_0}{D}$$



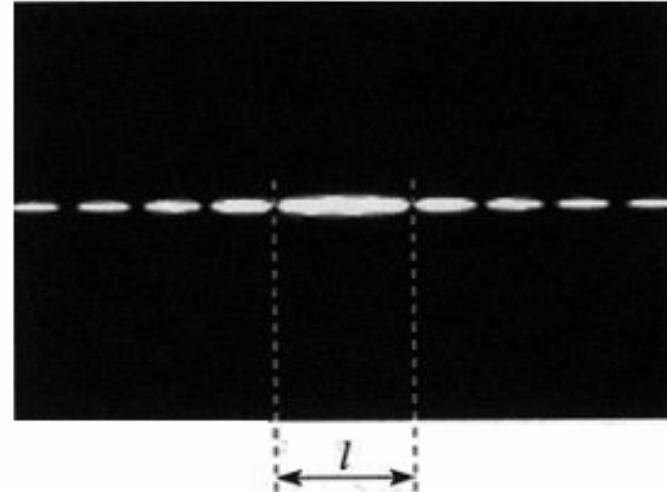
The Rayleigh criterion specifies that two point sources can be considered to be resolvable if the separation of the two images is at least the radius of the Airy disk (the first minimum of one coincides with the maximum of the other)

instrument	$D$	$\lambda$	$\Delta\theta$
lunette	14 cm	550 nm	$1''$
télescope (Mt Palomar)	5 m	550 nm	$(2,7 \cdot 10^{-2})''$
radiotélescope (Jodrell Bank)	75 m	21 cm	$700''$
œil humain	2 mm	550 nm	$70''$

## Diffraction par une fente fine : (Fraunhofer Diffraction by a slit aperture)



*Diffraction d'un faisceau laser par une fente fine.*



*Diffraction d'un faisceau laser par une fente.*

$$\ell \approx 2 \frac{\lambda_0 d}{a}$$

$$\theta_1 \approx \frac{\lambda_0}{a}$$

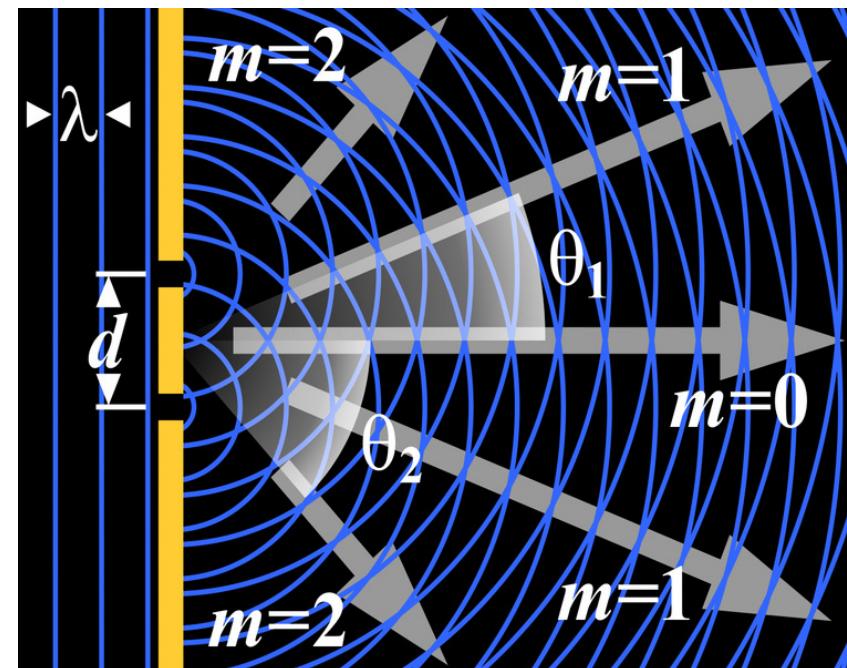
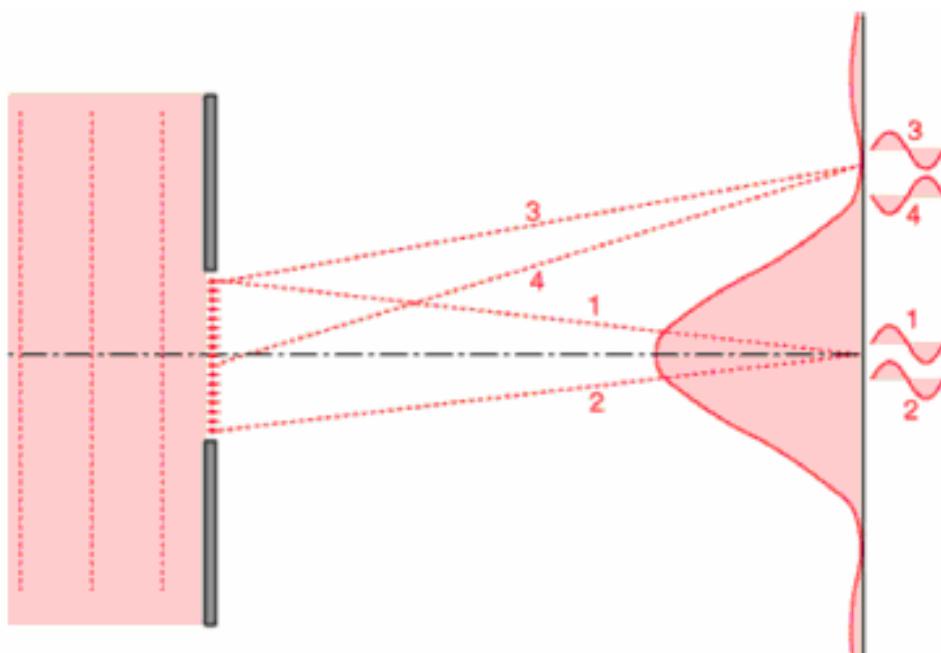
(half-angular length)

Demi - largeur angulaire :

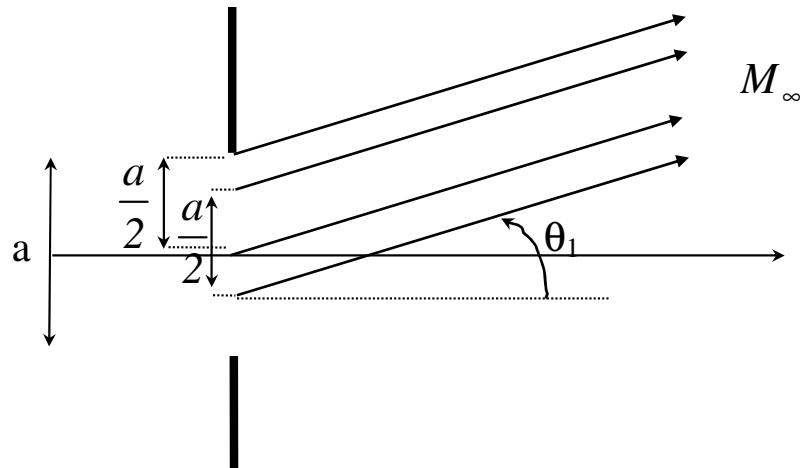
## Interprétation qualitative : (Single slit diffraction, qualitative explanation)

The phenomenon of diffraction involves the spreading out of waves past openings which are on the order of the wavelength of the wave.

The spreading of the waves into the area of the geometrical shadow can be modeled by considering small elements of the wavefront in the slit and treating them like point sources.



- Maximum de lumière (maximum of light, geometrical pattern) :  $\theta = 0$
- 1<sup>st</sup> light minimum : (destructive interference)



The light from a source located at the top edge of the slit interferes destructively with a source located at the middle of the slit when the path length difference is :

$$\frac{a}{2} \sin \theta_l \approx \frac{a}{2} \theta_l = \frac{\lambda_0}{2} \quad \Rightarrow \quad \theta_l = \pm \frac{\lambda_0}{a}$$

Similarly, the source just below the top of the slit will interfere destructively with the source located just below the middle of the slit at the same angle.

2<sup>nd</sup> light minimum : (destructive interference)

$$\frac{a}{4} \sin \theta_2 \approx \frac{a}{4} \theta_2 = \frac{\lambda_0}{2} \quad \Rightarrow \quad \theta_2 = \pm 2 \frac{\lambda_0}{a}$$

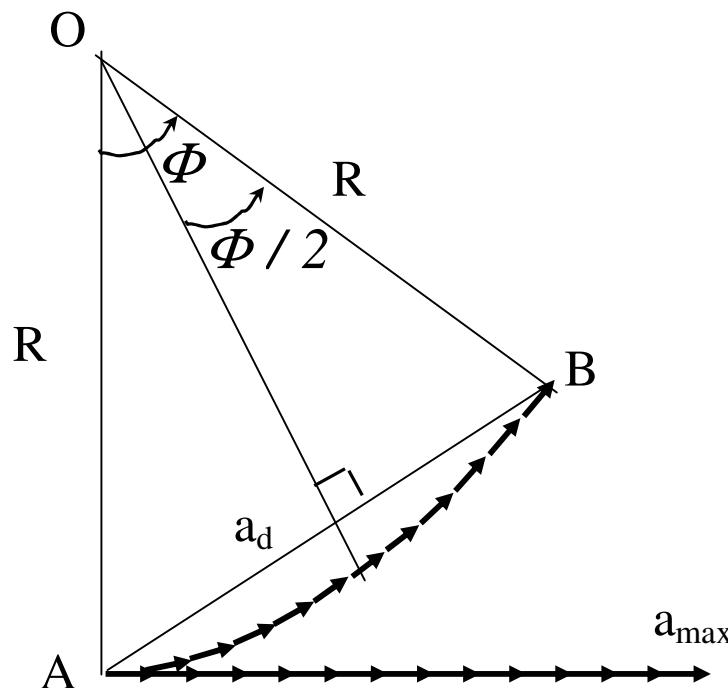
- A similar argument can be used to show that if we imagine the slit to be divided into six, eight parts, ...: minima are obtained at angles given by :

$$\theta_n = \pm n \frac{\lambda_0}{a} \quad (n = 1, 2, 3, 4, \dots)$$

## Calcul géométrique de l'amplitude diffractée : (diffracted wave's amplitude)

$$\Delta\varphi = \frac{2\pi}{\lambda_0} \frac{a}{N} \sin \theta \quad ; \quad a_n = a_0 e^{j(n-1)\Delta\varphi} \quad (n = 1 \text{ to } N)$$

$$a_d(M) = AB = \sum_{n=1}^N a_0 e^{j(n-1)\Delta\varphi}$$



(Fresnel's vectors theory)

$$\Phi = N \Delta \varphi = \frac{2\pi}{\lambda_0} a \sin \theta \quad ; \quad AB = 2R \sin \left( \frac{\Phi}{2} \right) \quad ; \quad E_{max} = \overset{\circ}{AB} = R\Phi$$

$$a_d(M) = E_{max} \frac{\sin \left( \frac{\pi}{\lambda_0} a \sin \theta \right)}{\frac{\pi}{\lambda_0} a \sin \theta} = E_{max} \sin c \left( \frac{\pi}{\lambda_0} a \sin \theta \right)$$

$$I_d(M) = I_0 \sin c^2 \left( \frac{\pi}{\lambda_0} a \sin \theta \right)$$

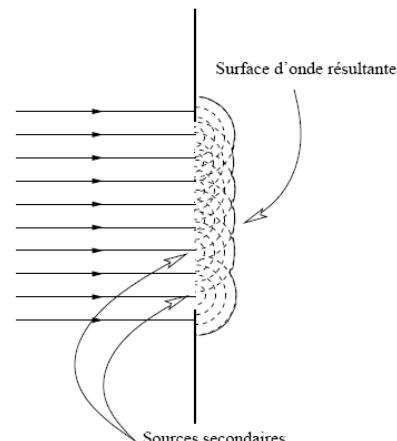
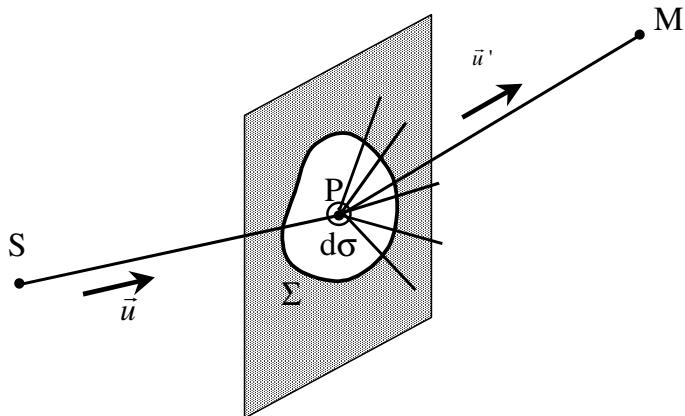
Minima in the diffracted light :

$$\frac{\pi}{\lambda_0} a \sin \theta_m = m\pi \quad \Rightarrow \quad \sin \theta_m \approx \theta_m = m \frac{\lambda_0}{a} \quad (m \in \mathbb{Z})$$

## Le principe de Huygens – Fresnel :

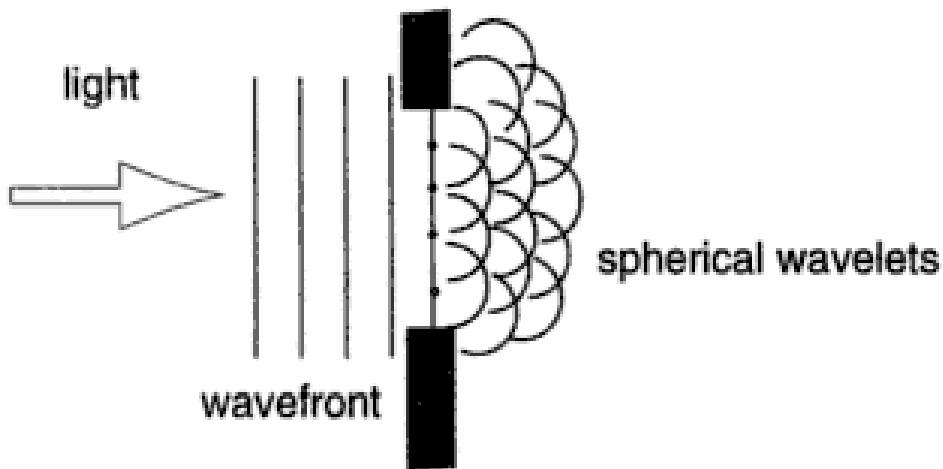
Soit  $(\Sigma)$  une ouverture plane éclairée par une source ponctuelle  $(S)$  monochromatique de longueur d'onde  $\lambda_0$ . Soit un découpage de  $(\Sigma)$  en éléments de surface  $d\sigma(P)$  centrés en  $P$ . Alors, pour le calcul de l'éclairement en un point  $M$  :

- Chaque élément de surface se comporte comme une source ponctuelle fictive, émettant une ondelette dont l'amplitude complexe instantanée en  $P$  est proportionnelle à l'amplitude complexe instantanée  $a_S(P,t)$  de l'onde émise par  $S$  en  $P$  et à l'élément de surface  $d\sigma(P)$ .
- Les sources fictives sont cohérentes : les ondes émises par ces sources secondaires interfèrent donc entre elles.



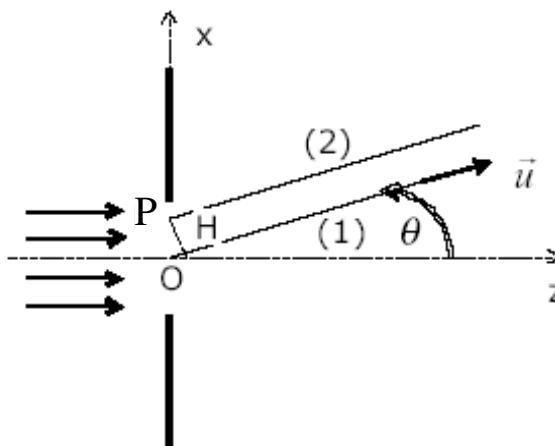
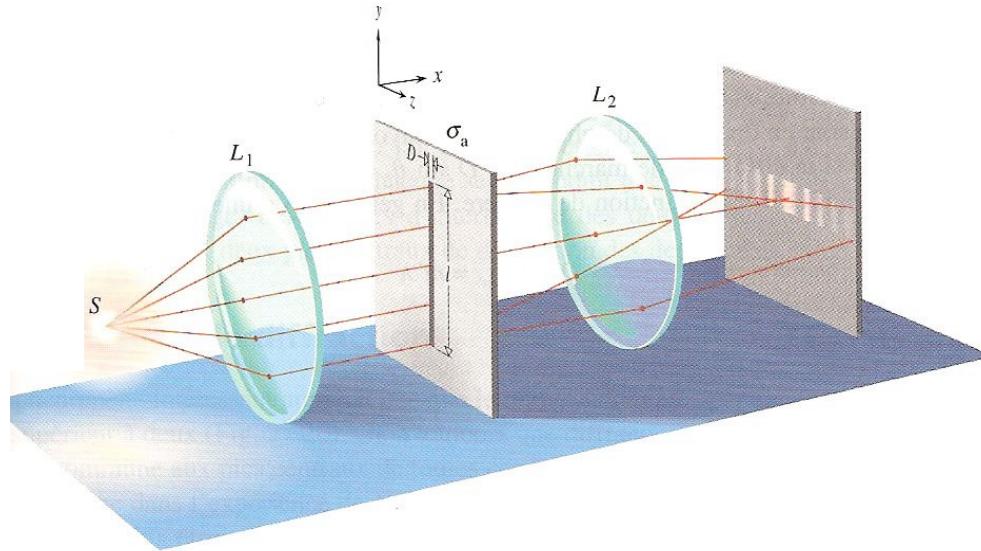
## Huygens – Fresnel principle :

- Each element of a wavefront of light may be considered to be the centre of a secondary disturbance which gives rise to a spherical wavelet.
- The position of the wavefront at any time is given by the envelope of all such wavelets
- The frequency and speed of the wavelets are the same as those of the primary waves
- The amplitude at any later point is the superposition of the wavelets.



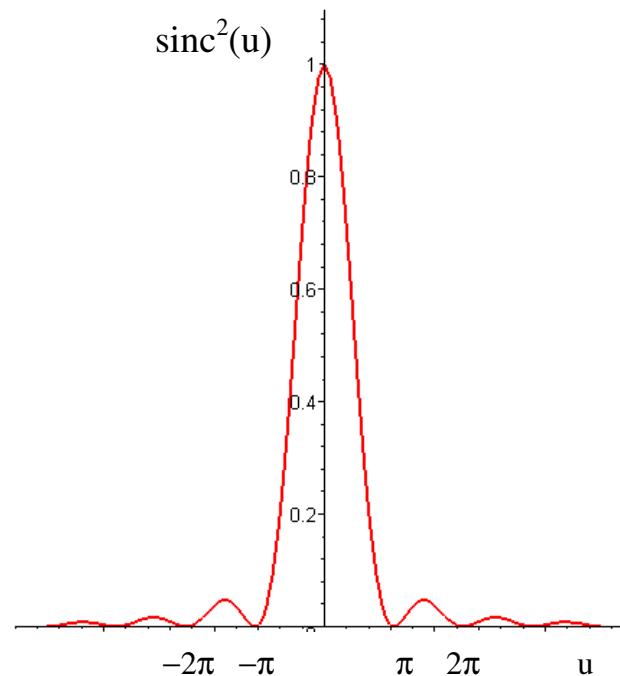
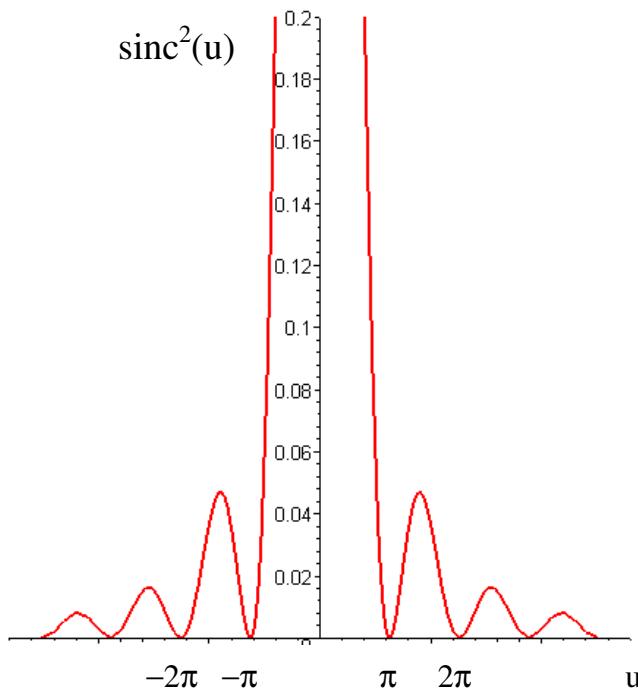
*The Huygens – Fresnel principle considers light diffraction as a summation of spherical waves.*

## Interprétation quantitative : (Single slit diffraction, quantitative explanation)



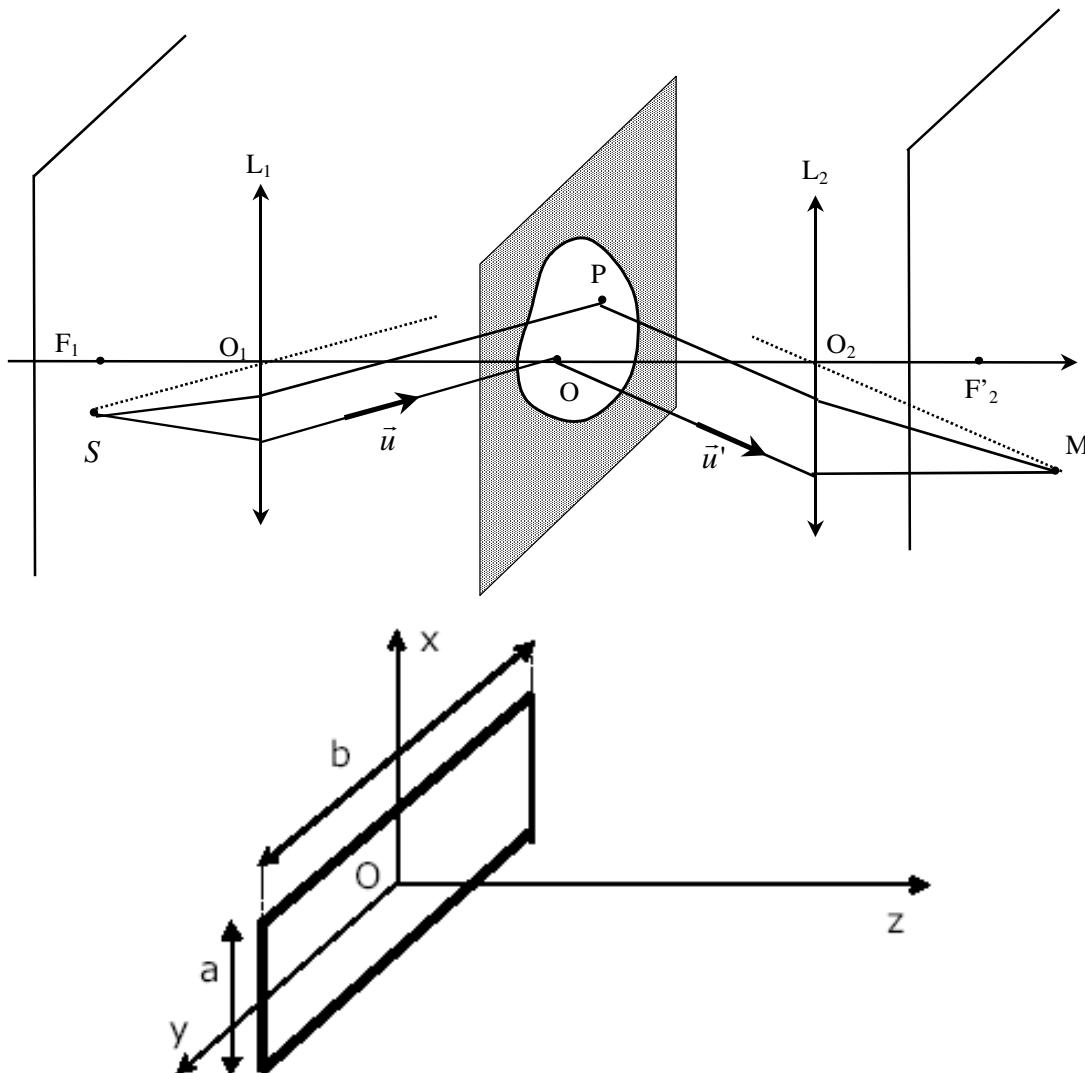
$$I(M) = I_0 \sin c^2 \left( \frac{\pi x a}{\lambda_0 f} \right) = I_0 \sin c^2(u)$$

$$(u = \frac{\pi x a}{\lambda_0 f})$$



**Graph de la fonction  $\text{sinc}^2(u)$  (plot of  $\text{sinc}^2(u)$ )**

## Avec une pupille rectangulaire : (Diffraction by a rectangular aperture)

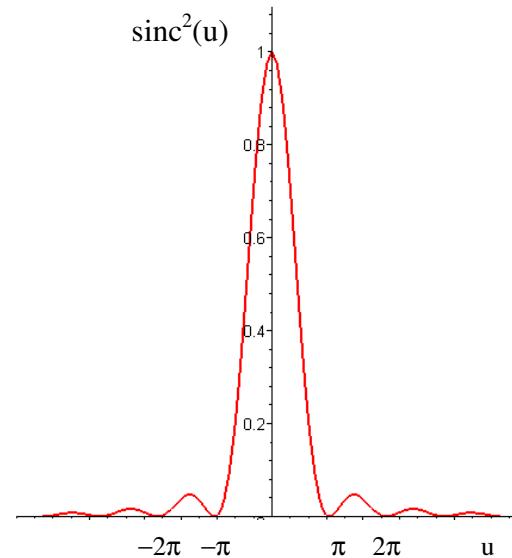
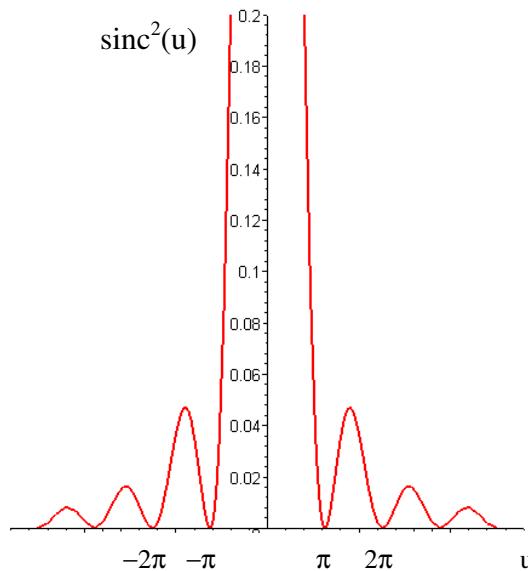


Amplitude diffractée : (diffracted amplitude)

$$\underline{a}(M) = KA_0 ab \exp[-ik_0(SOM)] \sin c\left(\frac{\pi(\alpha' - \alpha)a}{\lambda_0}\right) \sin c\left(\frac{\pi(\beta' - \beta)b}{\lambda_0}\right)$$

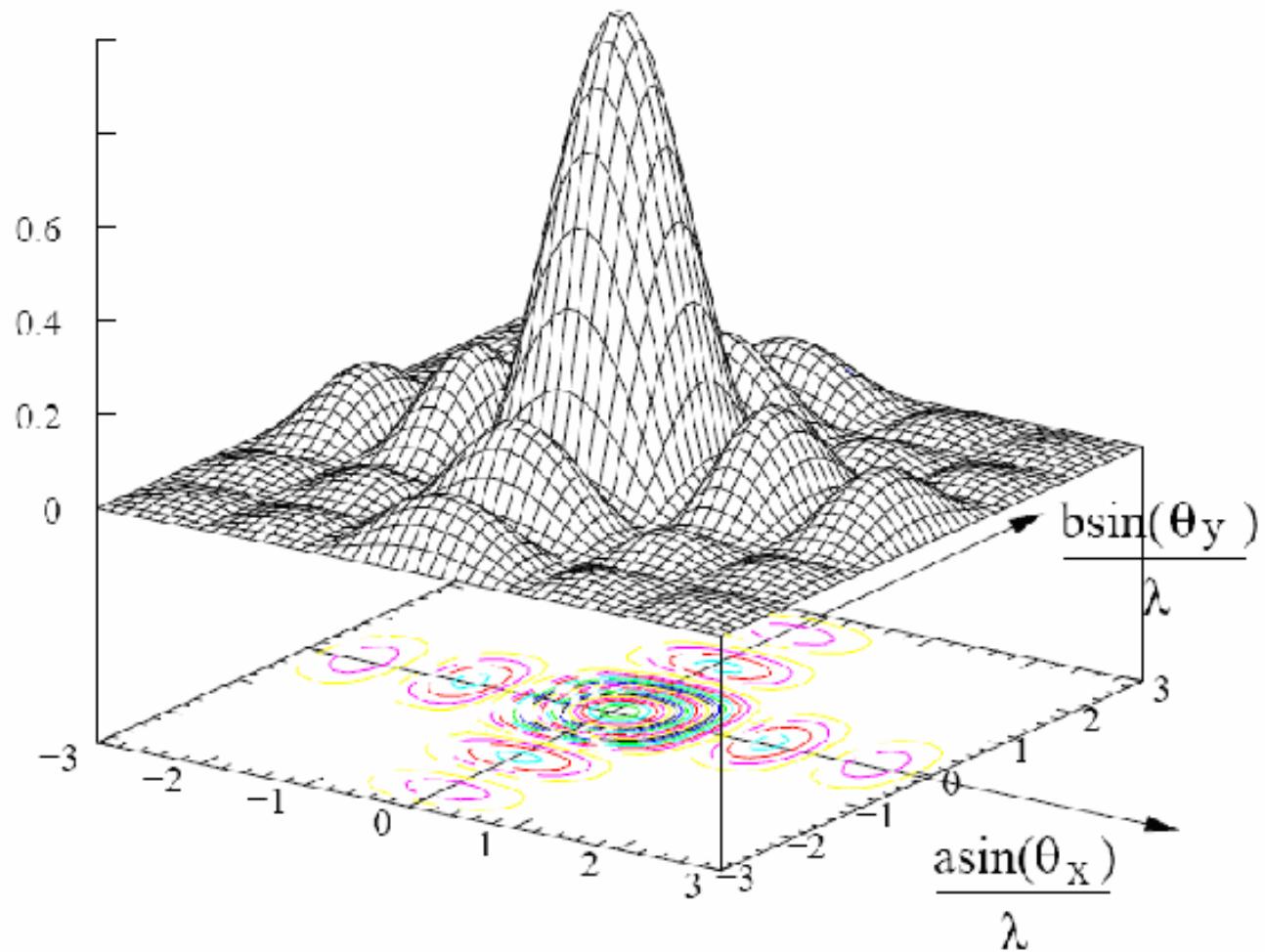
Eclaircissement (intensité) diffracté : (diffracted intensity)

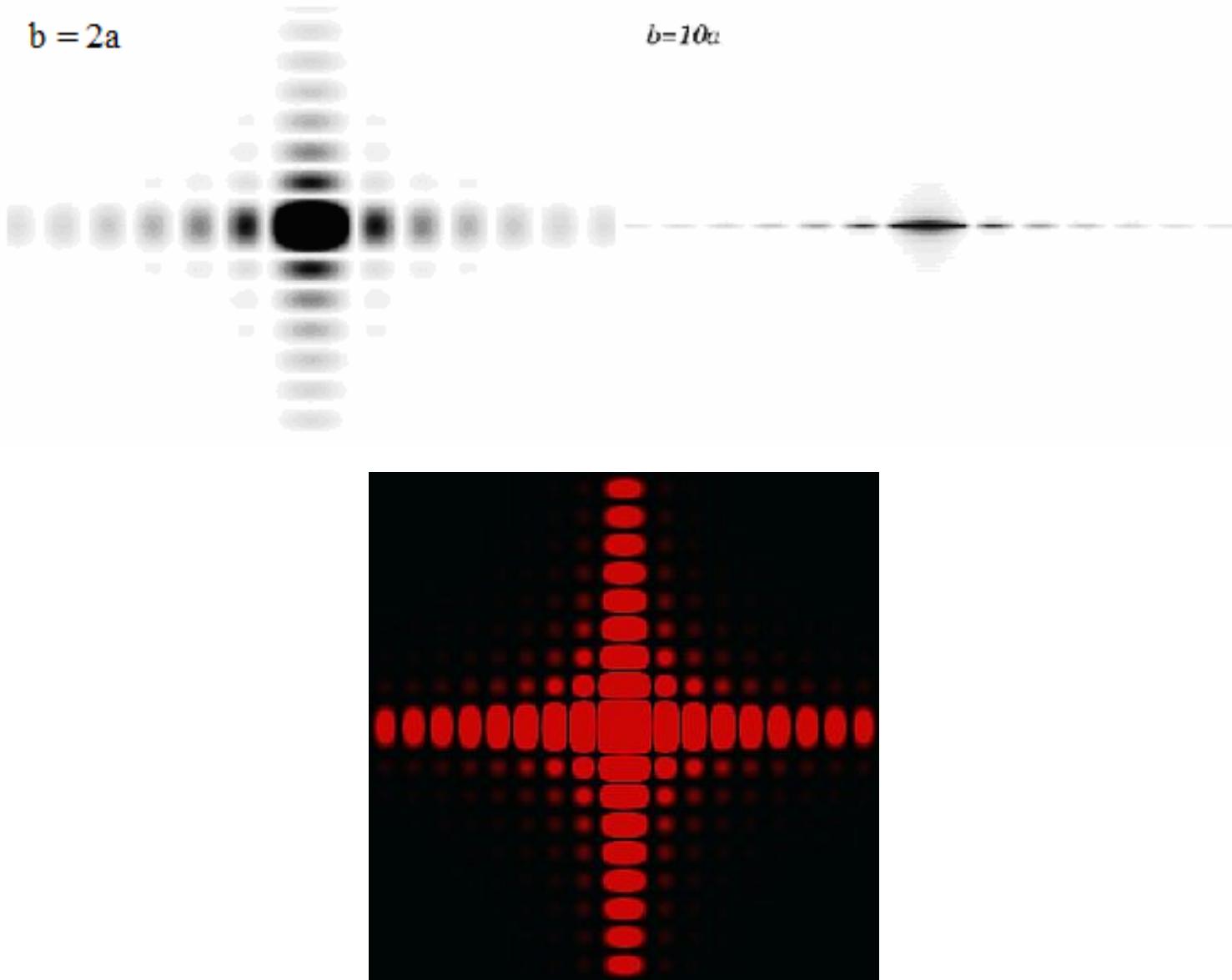
$$E(M) = E_0 \sin c^2\left(\frac{\pi(\alpha' - \alpha)a}{\lambda_0}\right) \sin c^2\left(\frac{\pi(\beta' - \beta)b}{\lambda_0}\right)$$

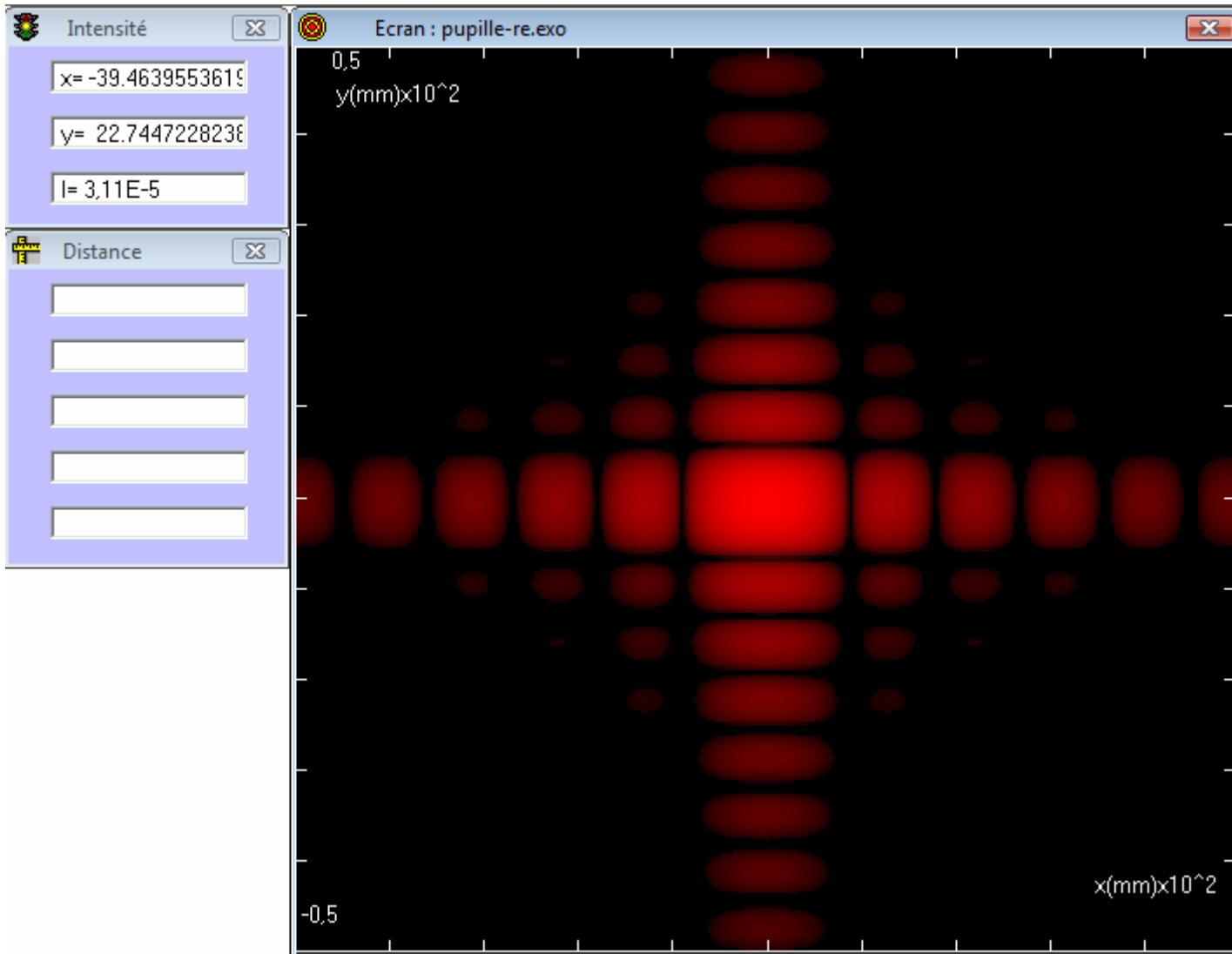


Graph de la fonction  $\text{sinc}^2(u)$  (plot of  $\text{sinc}^2(u)$ )

amplitude de l'onde diffractée par une fente avec  $a=b$







**File Diffint software : « Pupille rectangulaire » (rectangular aperture)**

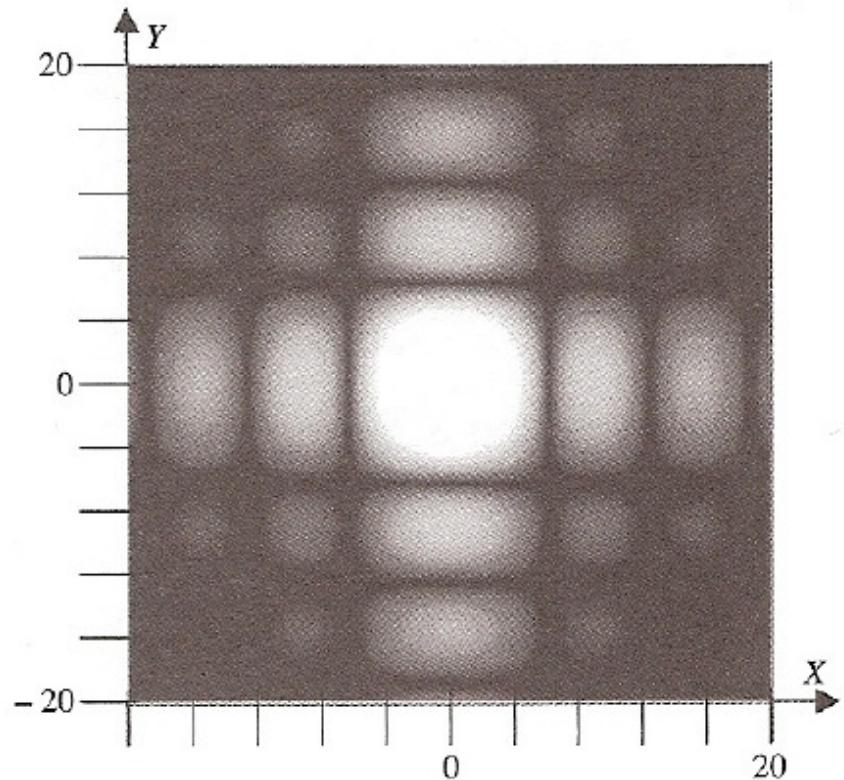
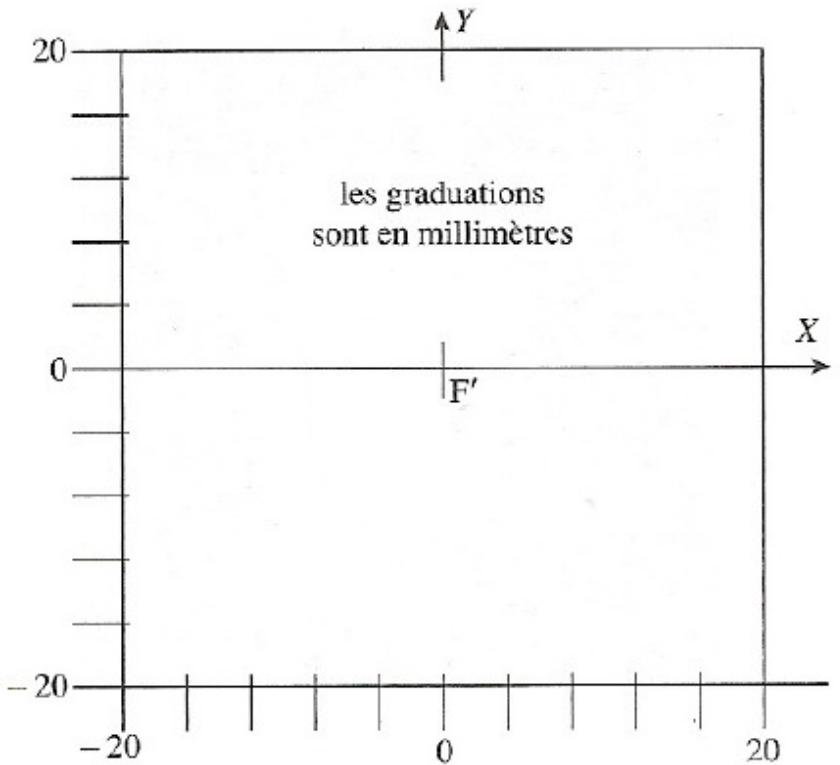


figure 1

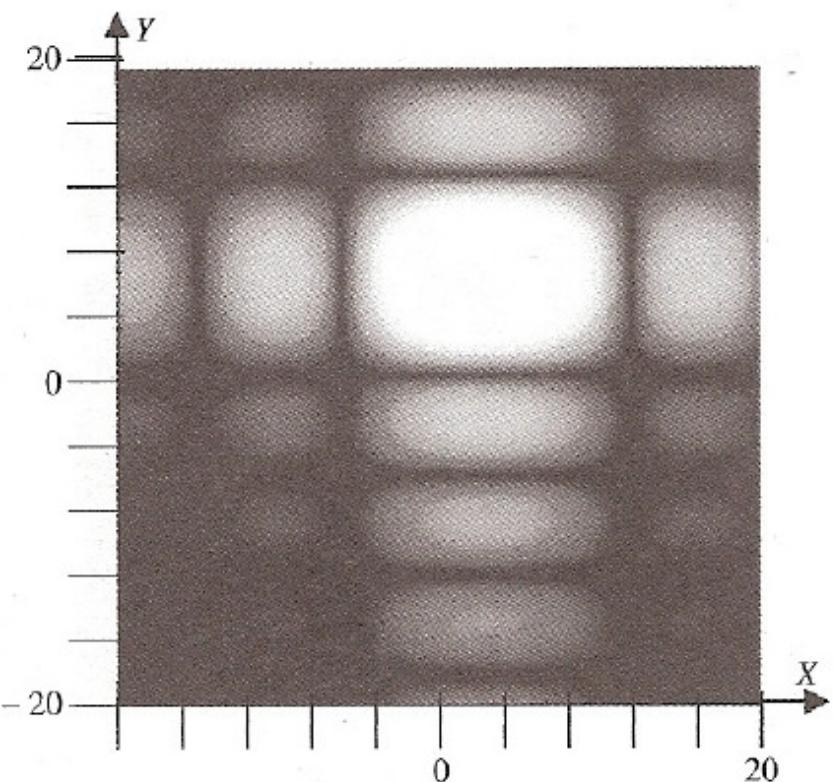


figure 4

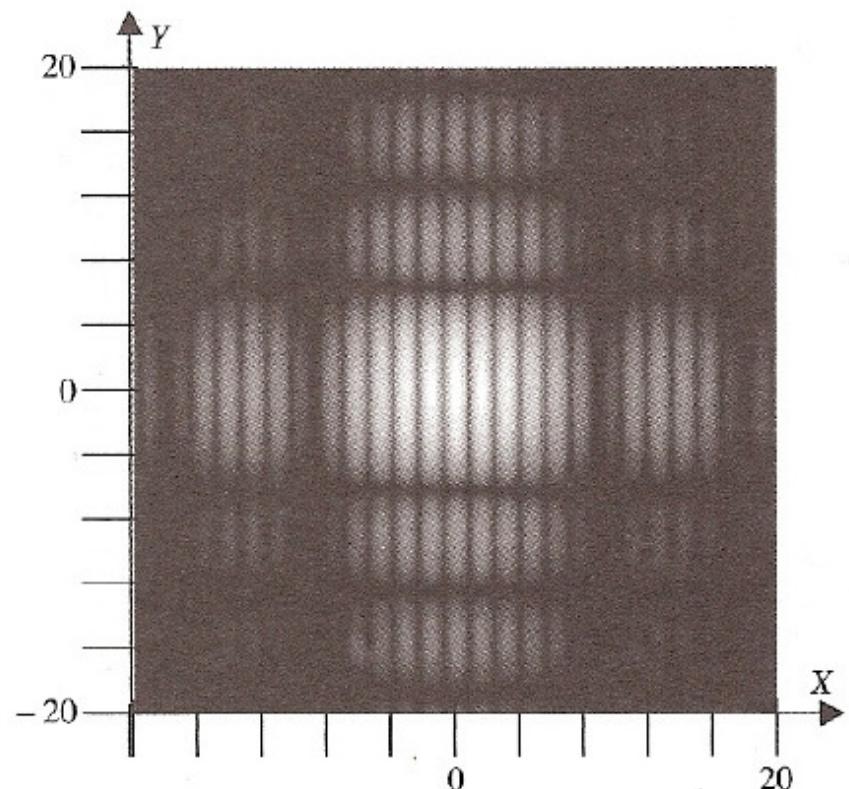


figure 5

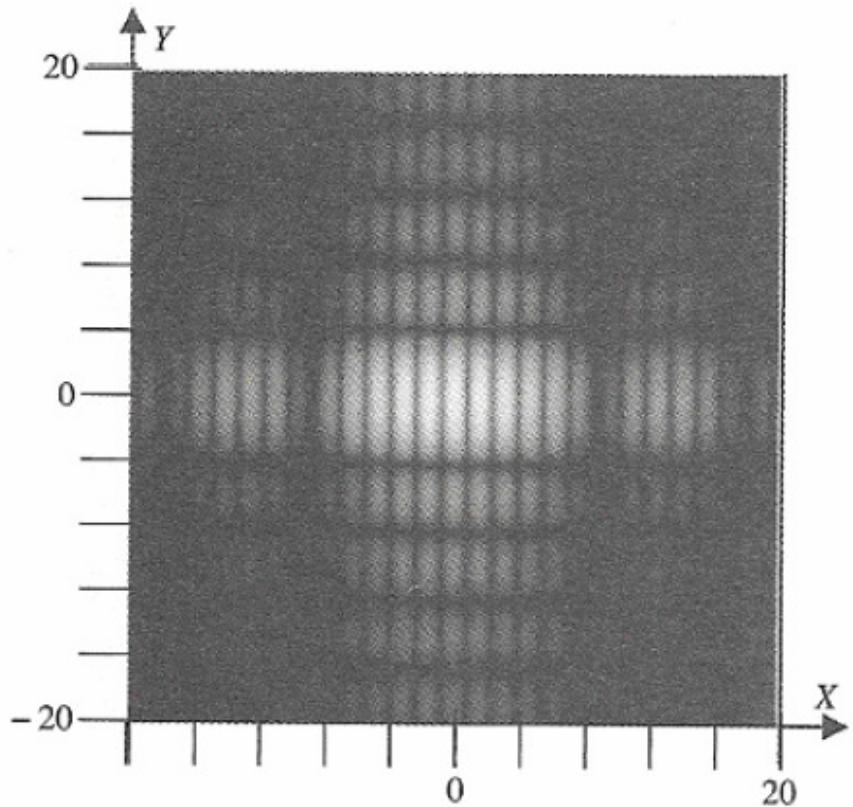


figure 6

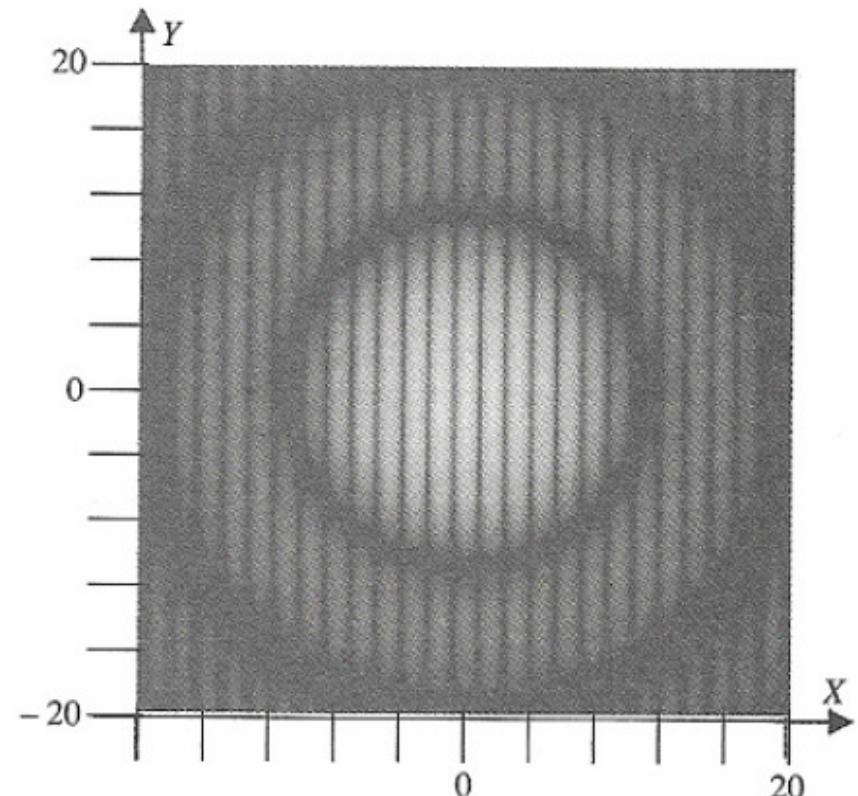


figure 7

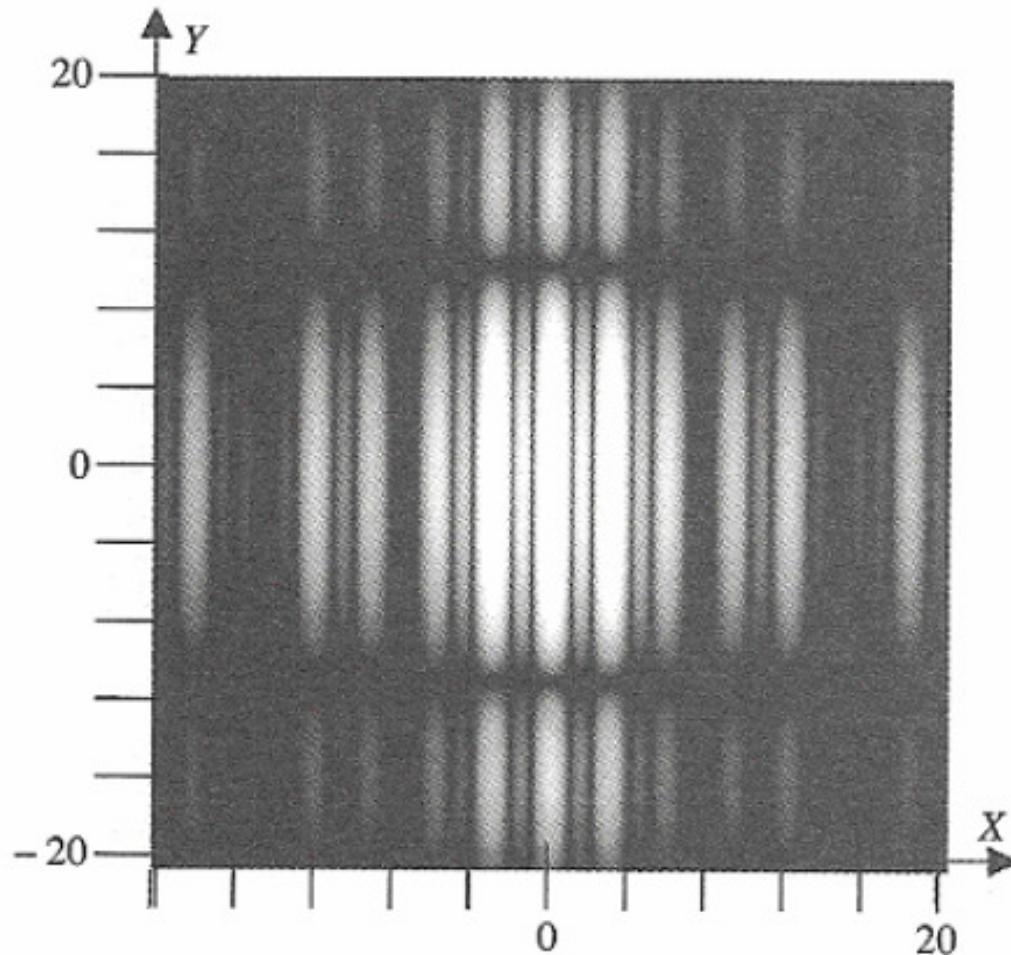
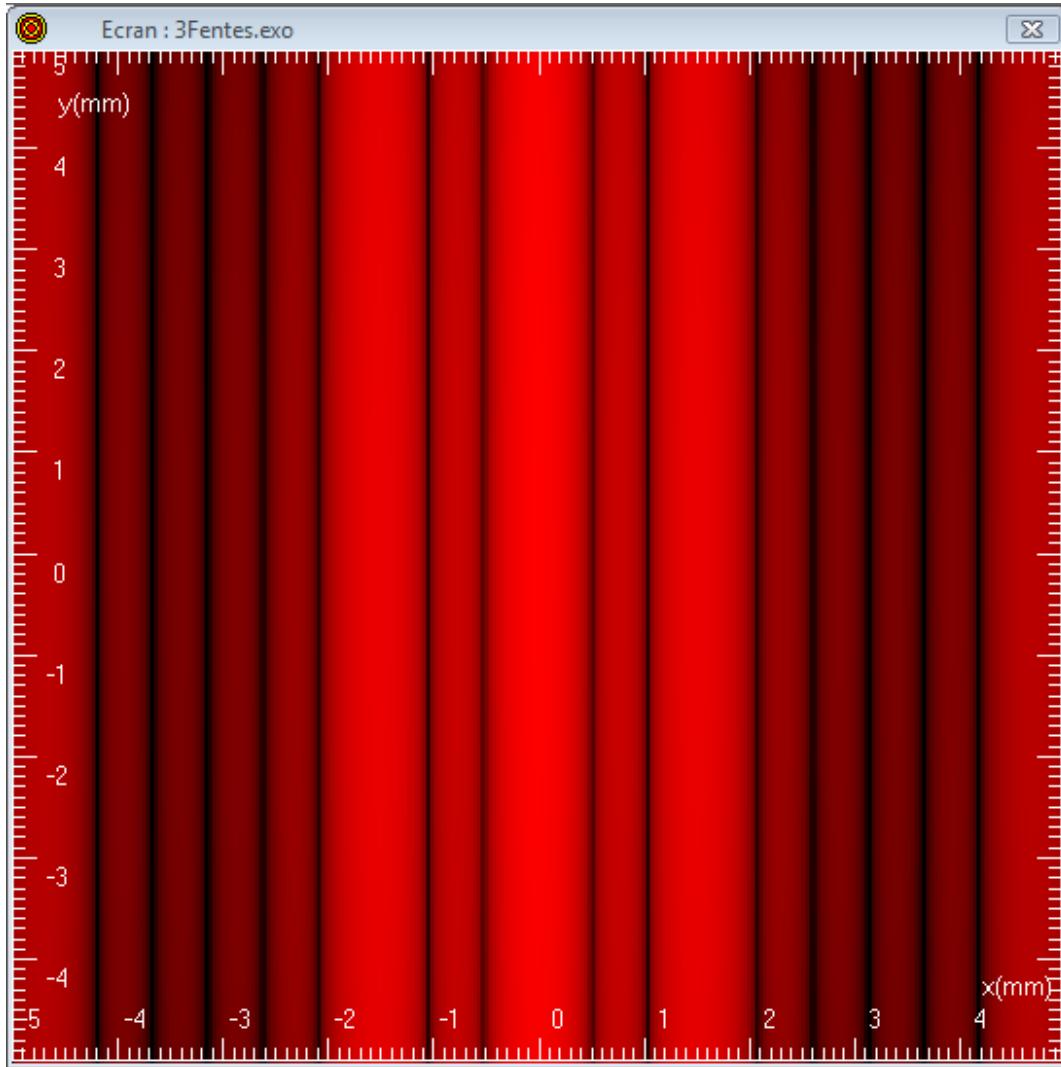
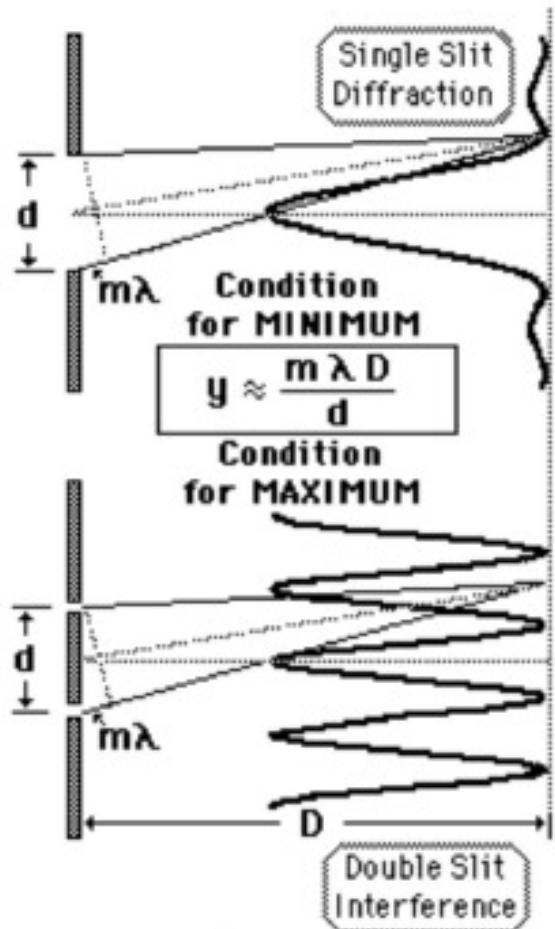


figure 10



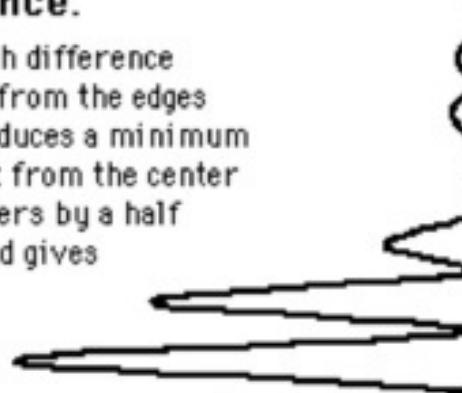
**File Diffint : « 3fentes »**

# Single and Double Slit Comparison



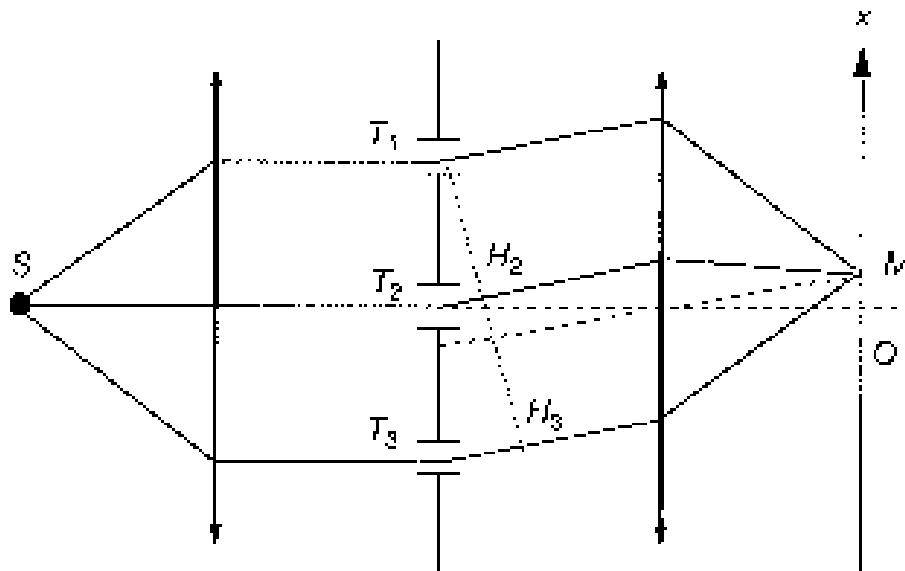
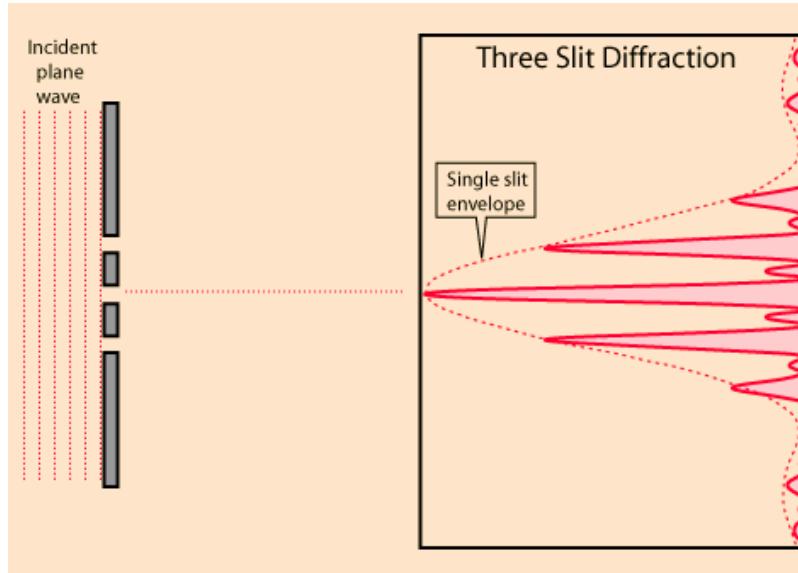
## Comparison of Single Slit Diffraction and Double Slit Interference.

One wavelength difference in pathlength from the edges of the slit produces a minimum since the light from the center of the slit differs by a half wavelength and gives destructive interference.

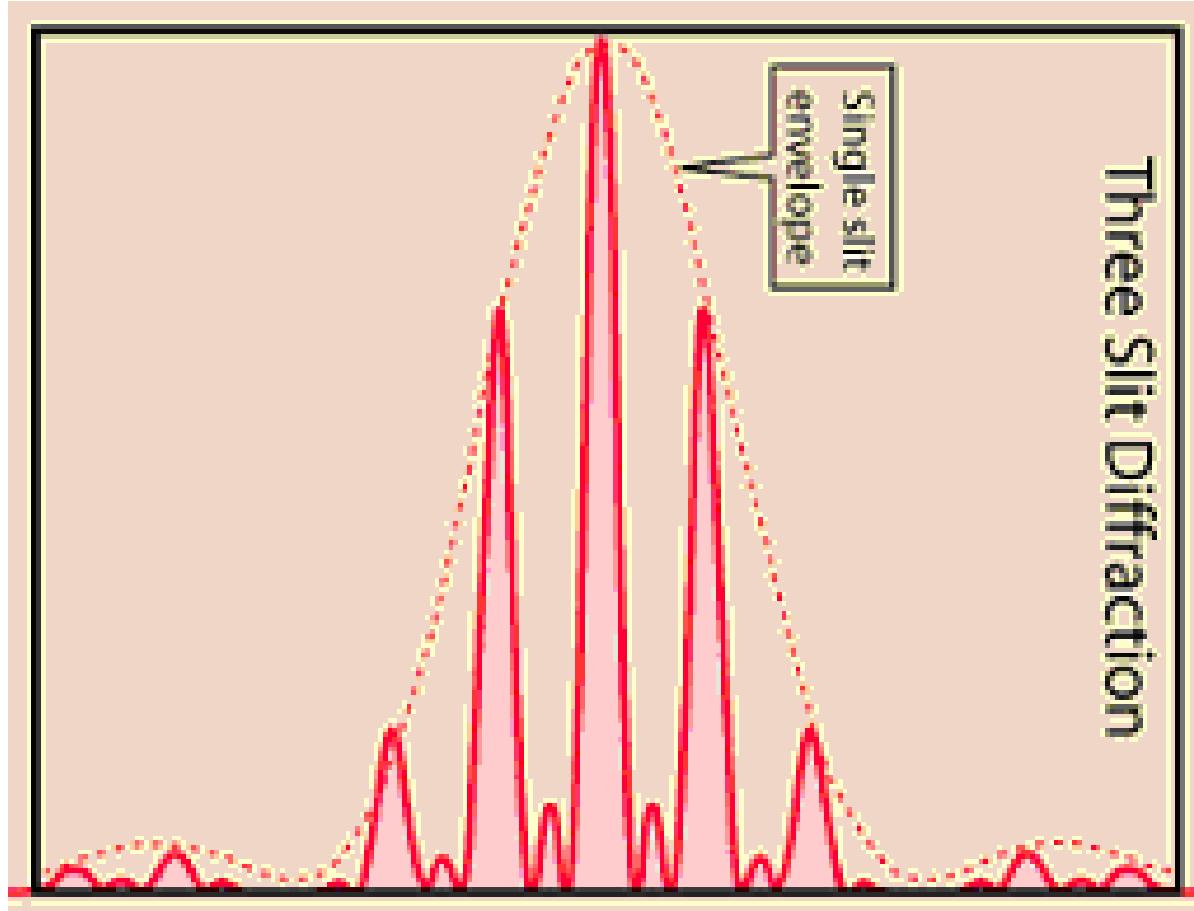


One wavelength difference in pathlength from the two slits produces constructive interference.

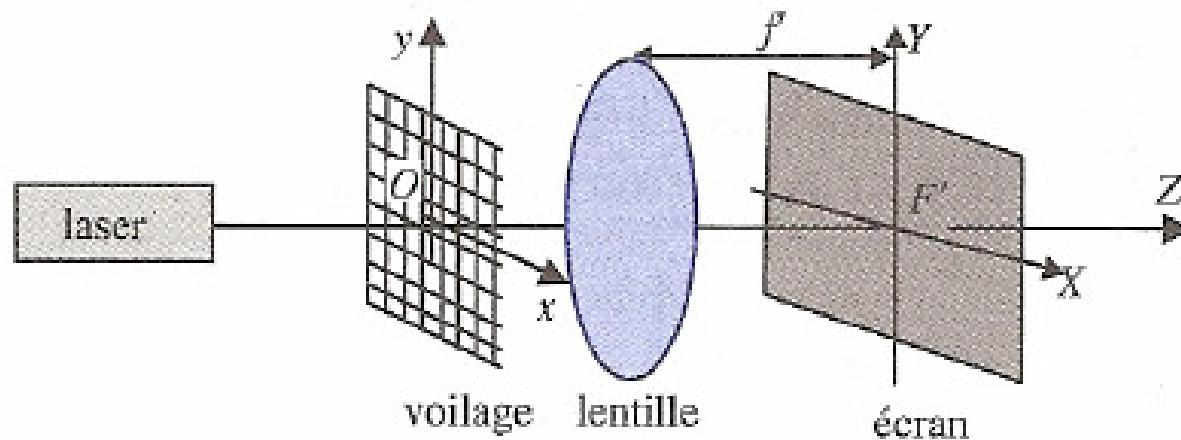
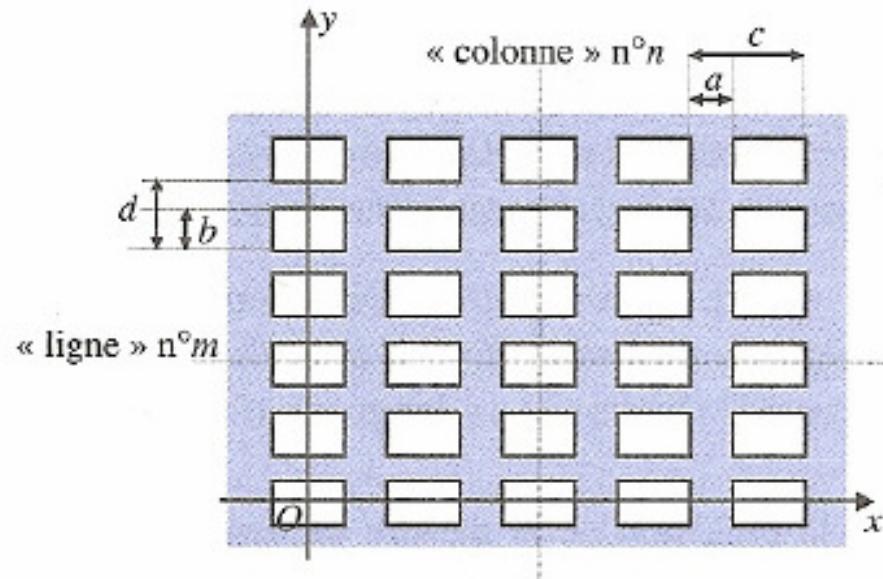
Combined Diffraction and Interference for a Double Slit.

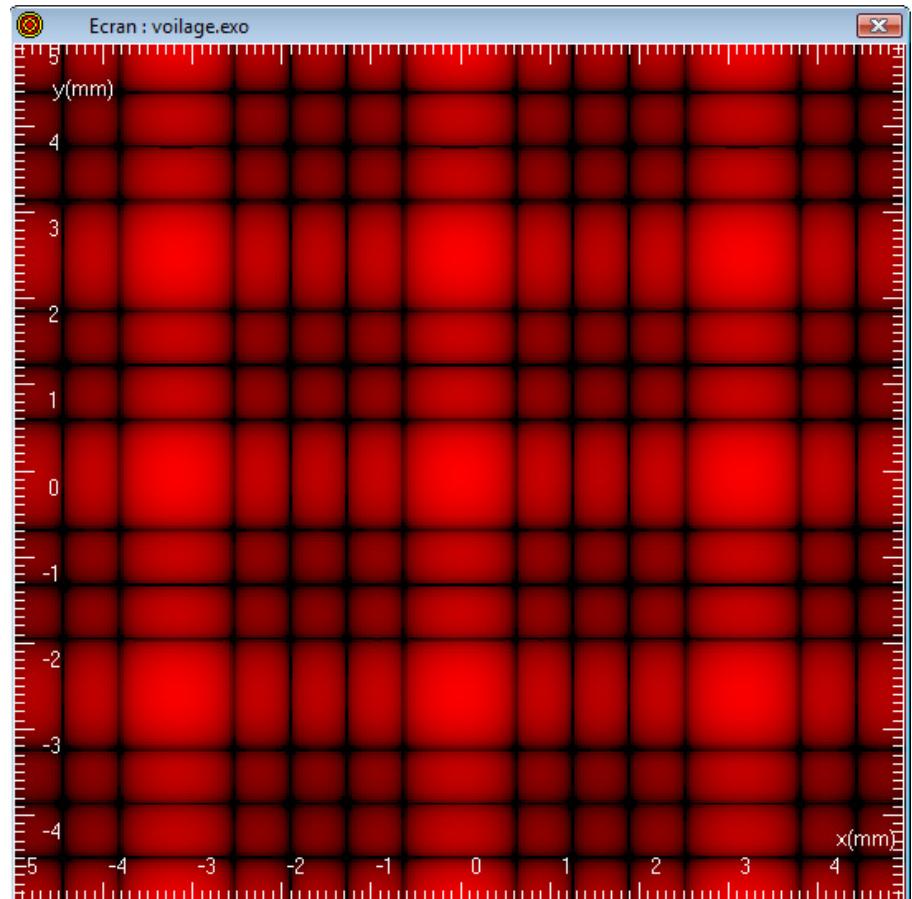


$$I = I_0 \sin c^2 \left( \frac{\pi a x}{\lambda_0 f'} \right) \left( 1 + 2 \cos \left( \frac{2\pi}{\lambda_0} \frac{dx}{f'} \right) \right)^2$$



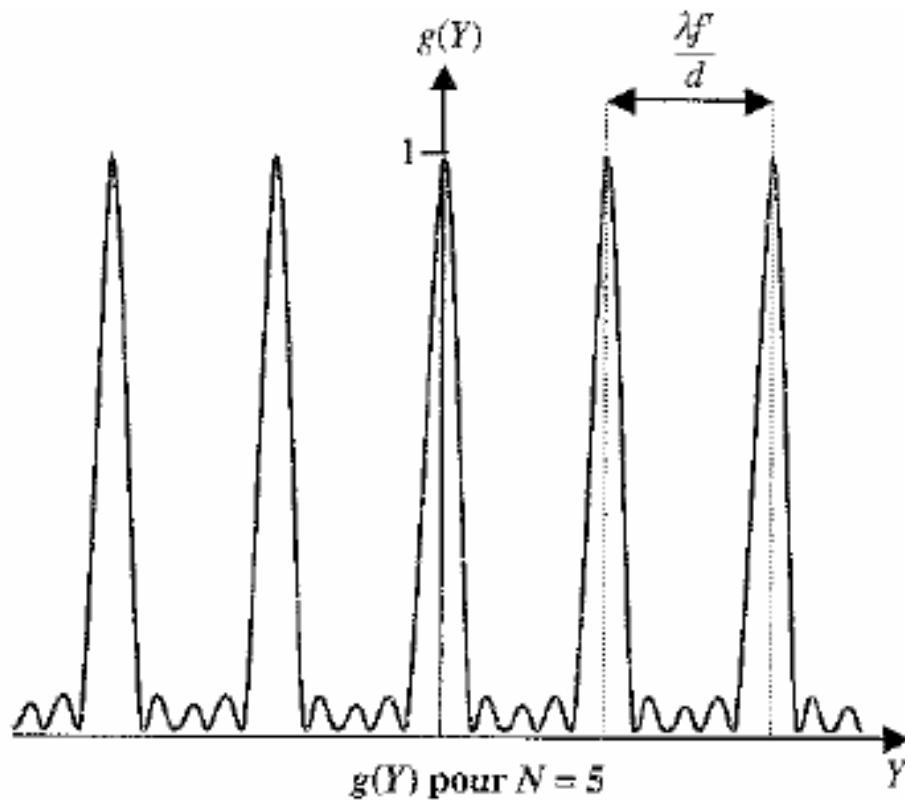
The aperture is a net curtain :





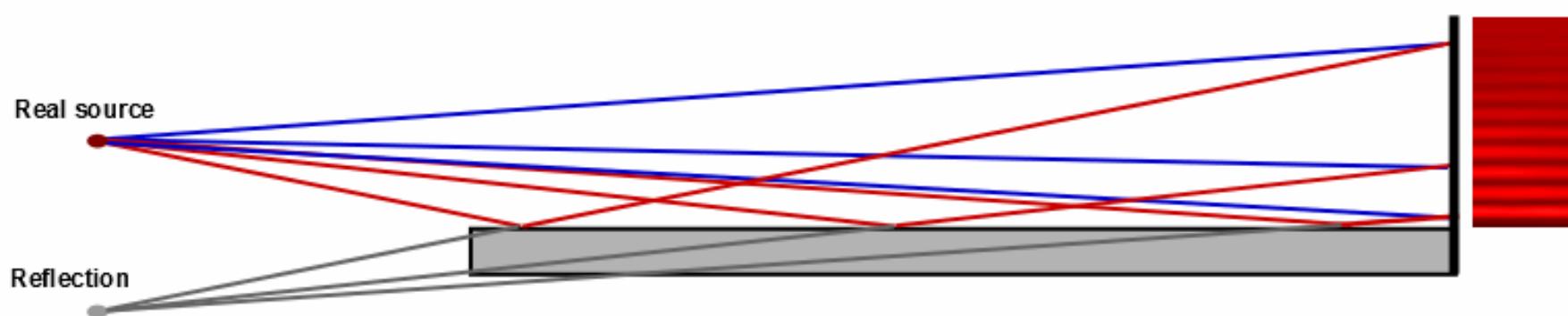
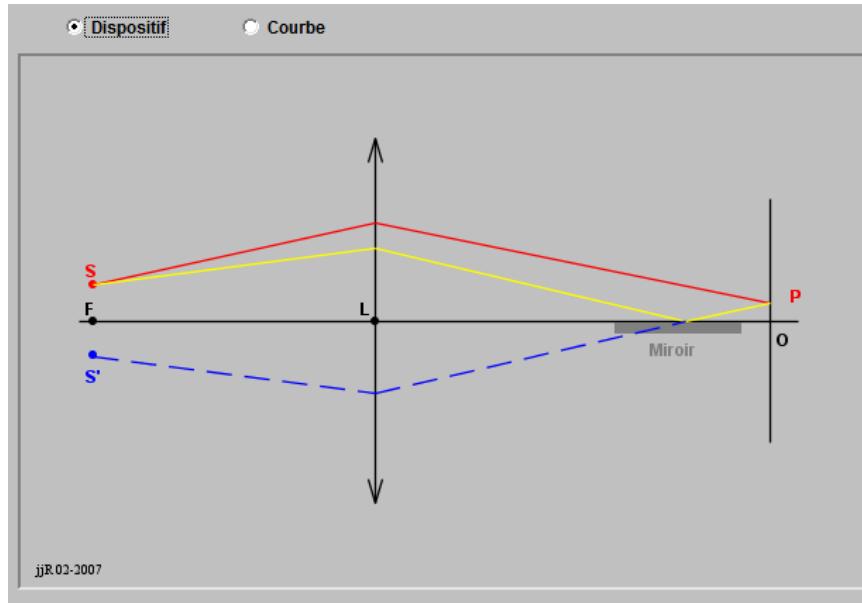
File Diffint : « [voilage.exo](#) »

$$I(x, y) = I_{max} \sin^2\left(\frac{\pi ax}{\lambda_0 f'}\right) \sin^2\left(\frac{\pi by}{\lambda_0 f'}\right) \left( \frac{\sin\left(\frac{N\pi cx}{\lambda_0 f'}\right)}{N \sin\left(\frac{\pi cx}{\lambda_0 f'}\right)} \right)^2 \left( \frac{\sin\left(\frac{M\pi dy}{\lambda_0 f'}\right)}{M \sin\left(\frac{\pi dy}{\lambda_0 f'}\right)} \right)^2$$



# Animation JJR : le miroir de Lloyd (Lloyd's mirror)

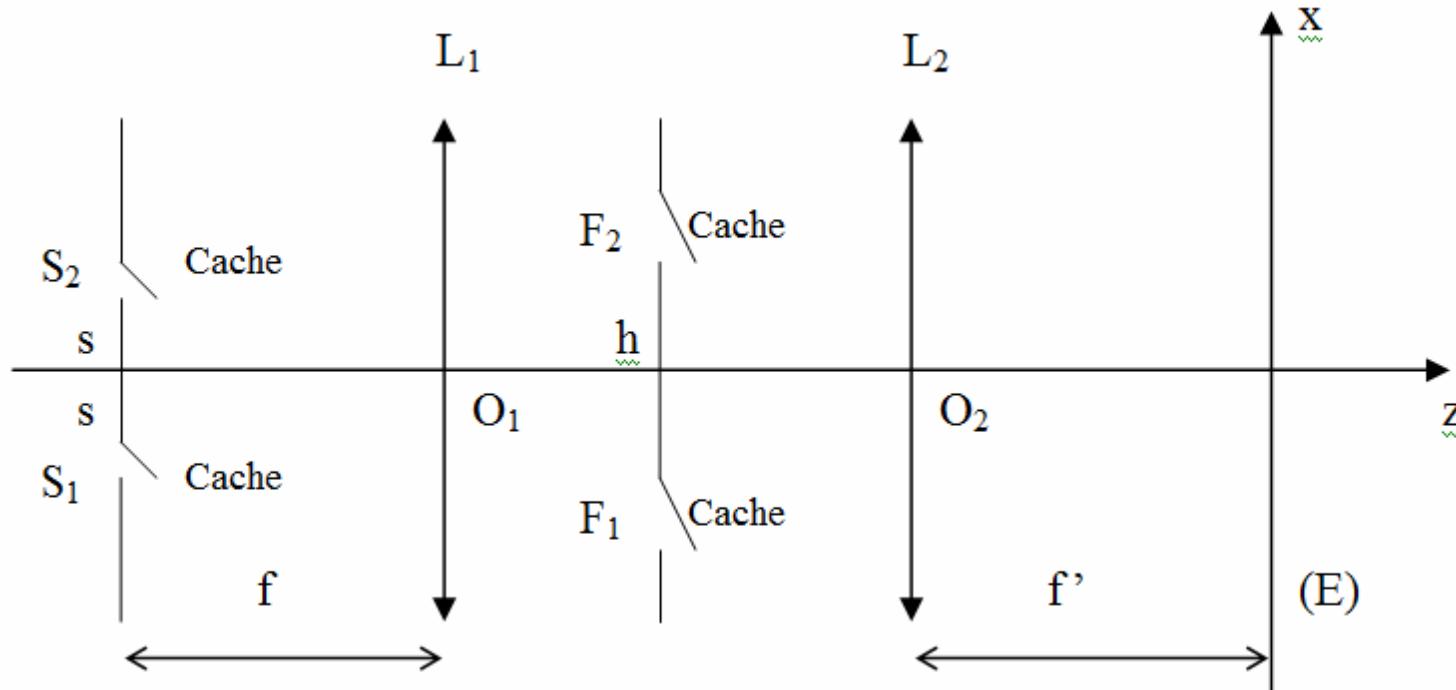
## Animations JJR/Optique ondulatoire/Interférences/Miroir de Lloyd



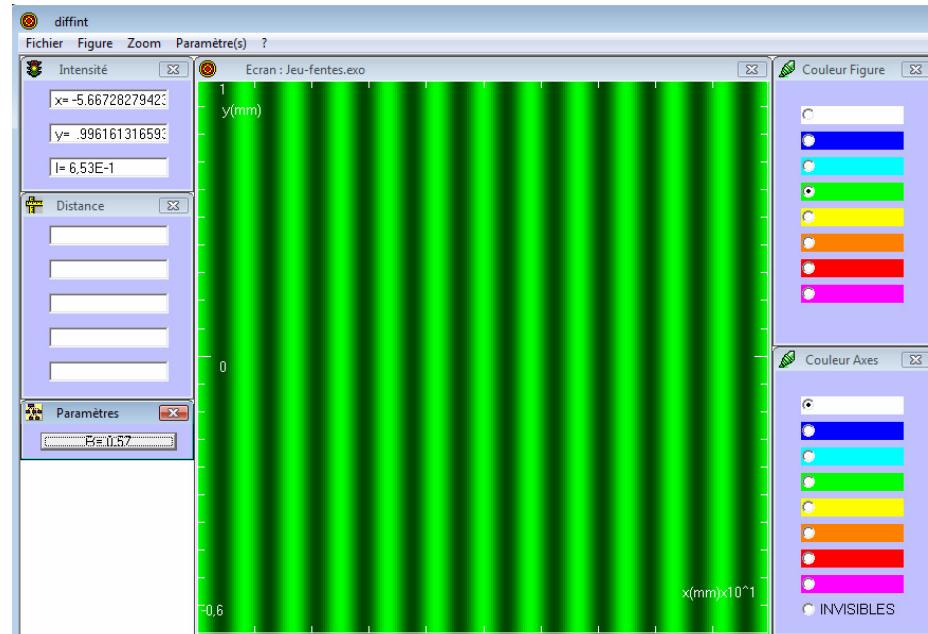
Deux sources ponctuelles, situées dans le plan focal objet de  $L_1$  de focale  $f$ , monochromatiques de même longueur d'onde  $\lambda$  sont espacées de  $2s$ .

Derrière  $L_1$ , on place deux fentes identiques, très longues dans la direction perpendiculaire au plan de la feuille, de largeur  $a$  et séparées d'une distance  $h$ .

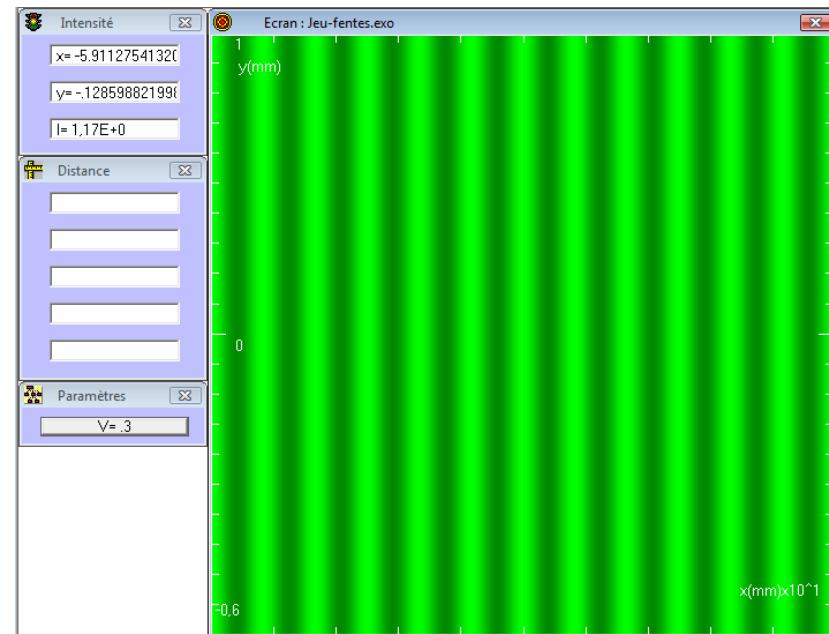
Enfin, on observe les figures d'interférences et/ou de diffraction sur un écran ( $E$ ) placé dans le plan focal image de la lentille  $L_2$ , de focale  $f'$ .



- 1) Combien y a t'il de fonctionnements possibles en jouant sur les caches des sources et des fentes ?
- 2) Dégrossir les cas évidents et restreindre au maximum l'étude.
- 3) Décrire, selon les différentes longueurs intervenant dans l'exercice et avec un minimum de calculs, les phénomènes observés sur l'écran (E).
- 4) Le fichier **Jeu-de-fentes.exo** donne l'allure de l'écran dans un des cas évoqués à la question (3). Lequel ? Quelle(s) longueur(s) peut-on déterminer ? On donne  $f = f' = 1 \text{ m}$ .



$$V = 0,57$$



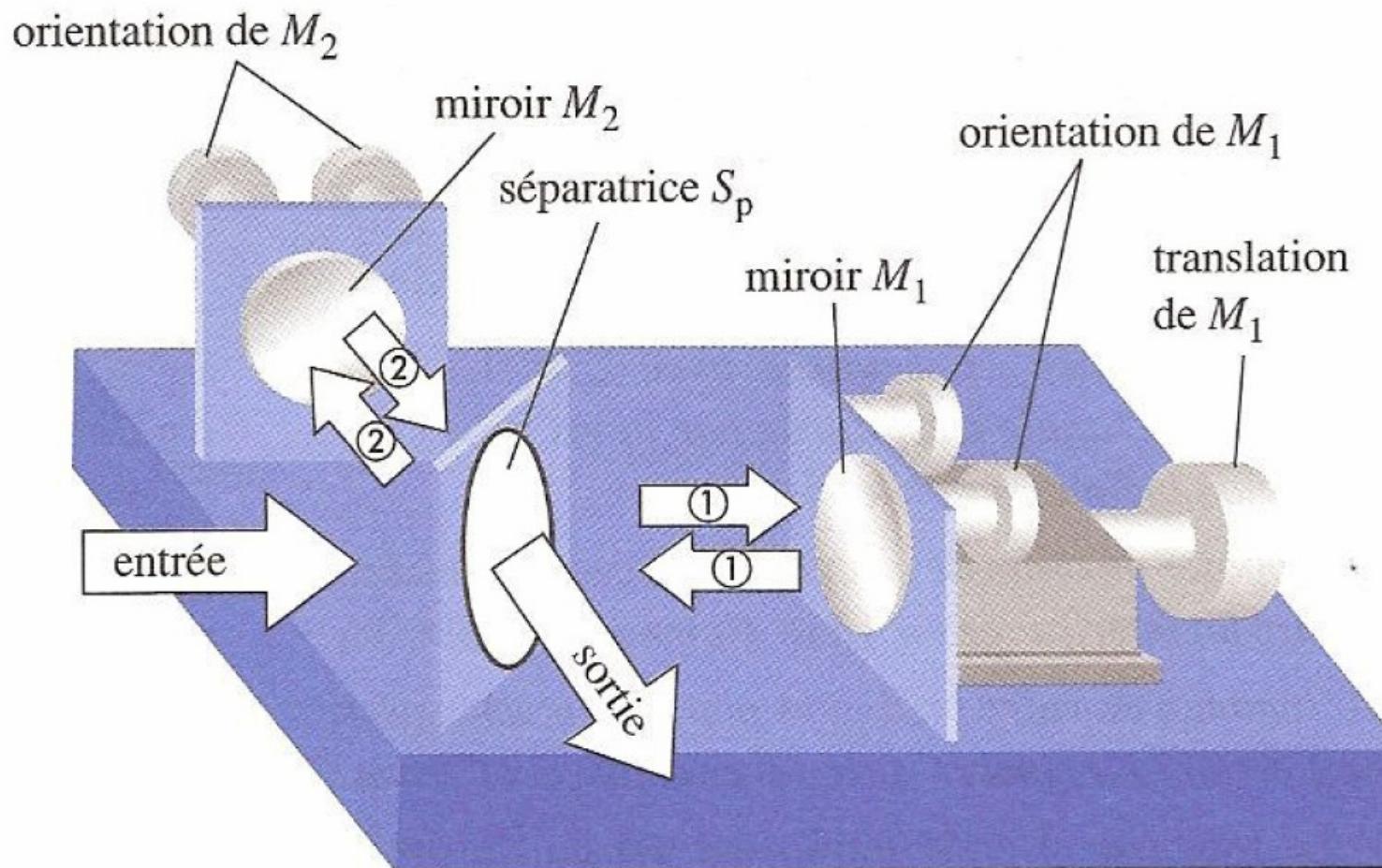
$$V = 0,3$$

**Cas de deux fentes sources séparées de 2s (la diffraction est isotrope) :**

Problème de cohérence spatiale :

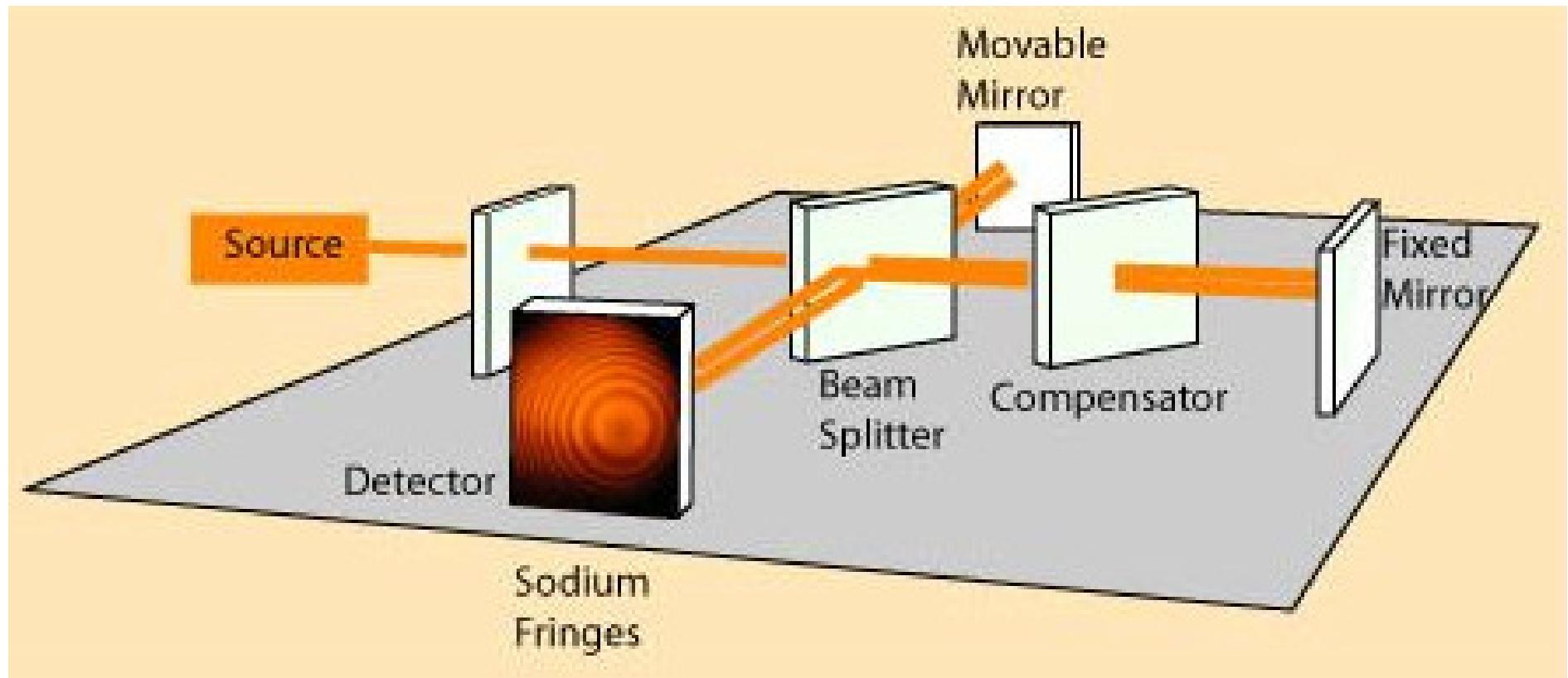
$$I(M) = 4I_0 \left( 1 + \cos\left(\frac{2\pi}{\lambda_0} \frac{a(2s)}{2f}\right) \cos\left(\frac{2\pi}{\lambda_0} \frac{ax}{f'}\right) \right) = 4I_0 \left( 1 + V \cos\left(\frac{2\pi}{\lambda_0} \frac{ax}{f'}\right) \right)$$

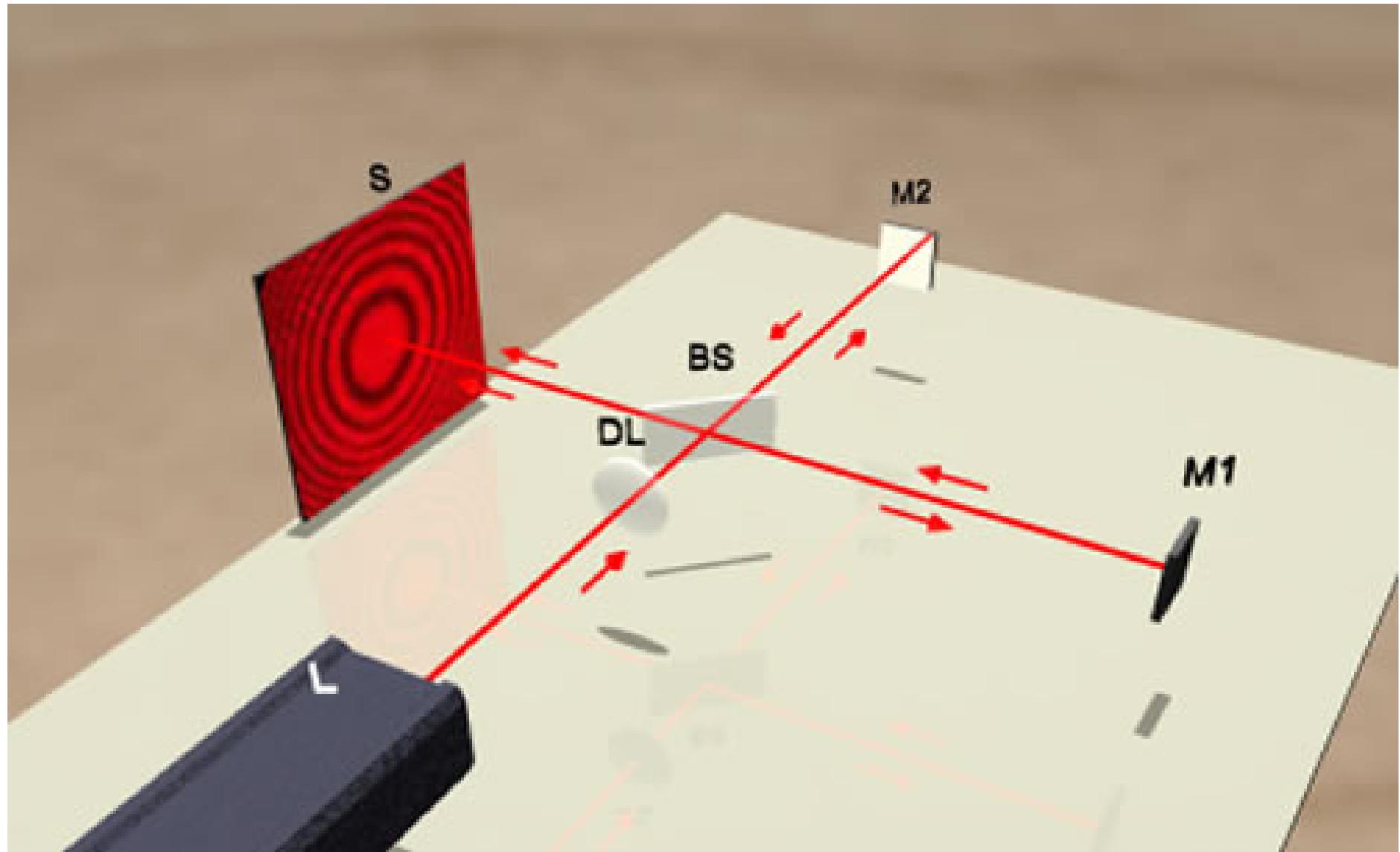
## Vidéo : « Le Michelson en lame d'air »



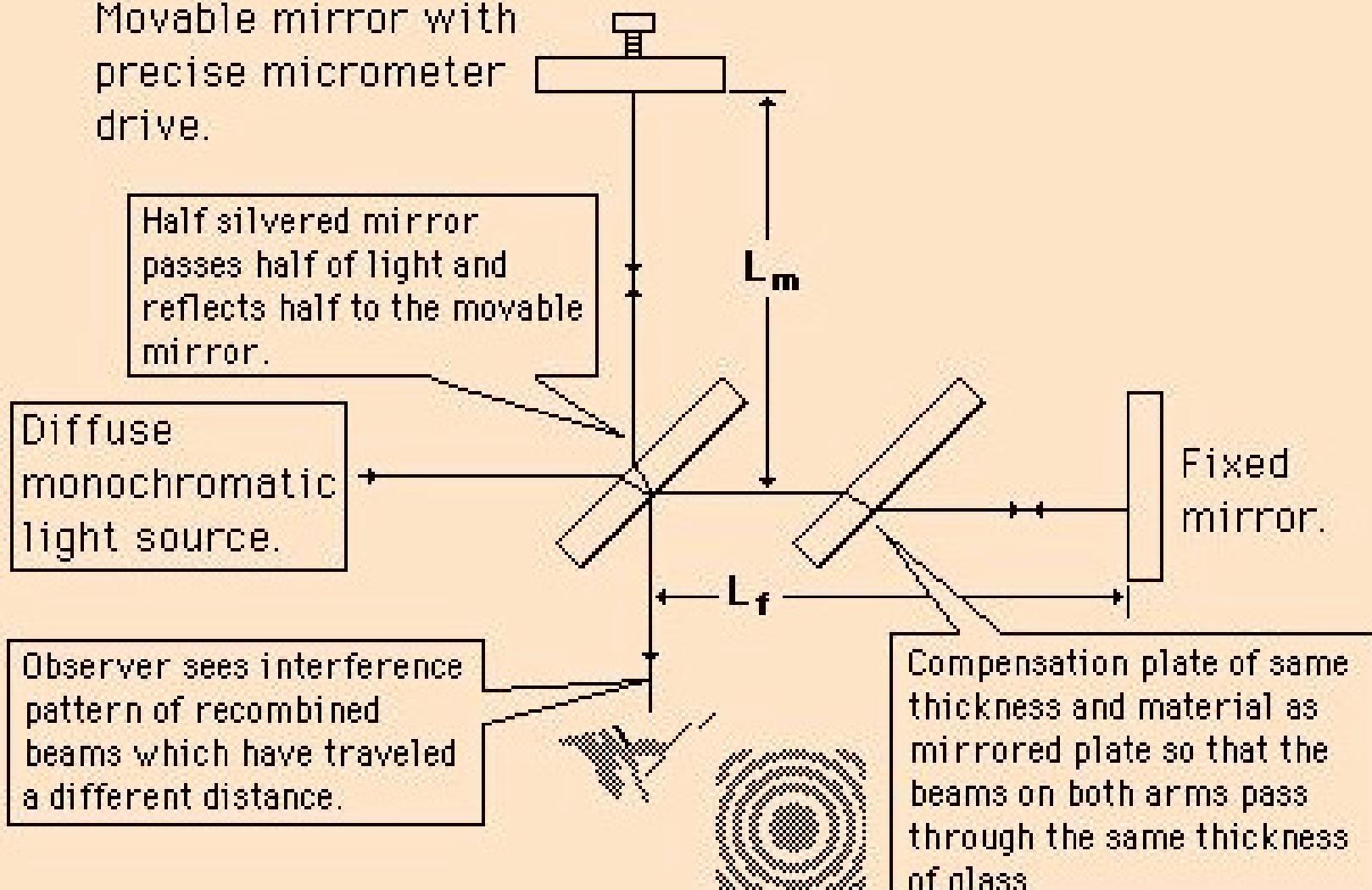
The Michelson interferometer produces interference fringes by splitting a beam of monochromatic light so that one beam strikes a fixed mirror and the other a movable mirror.

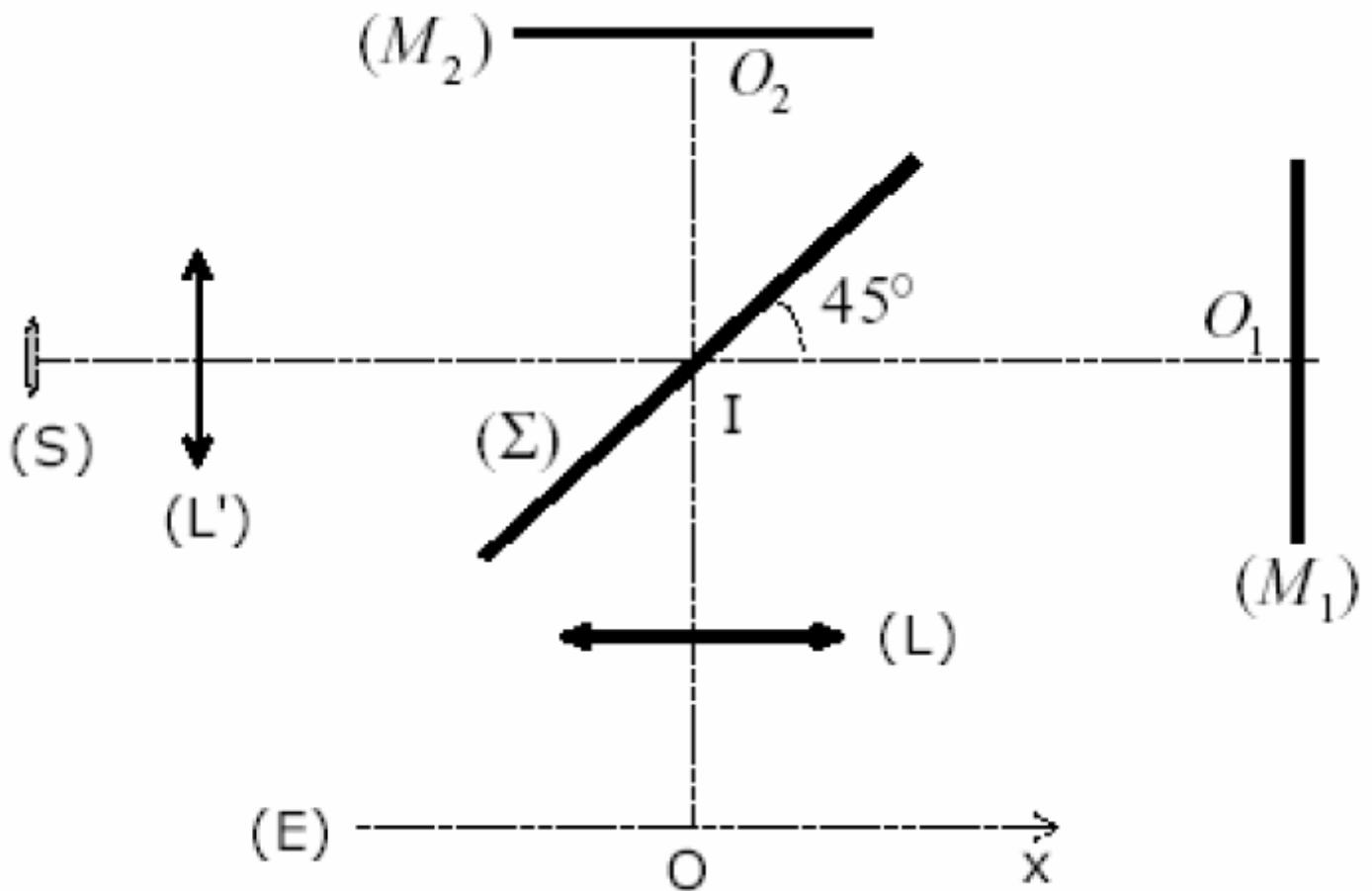
When the reflected beams are brought together, an interference pattern results.



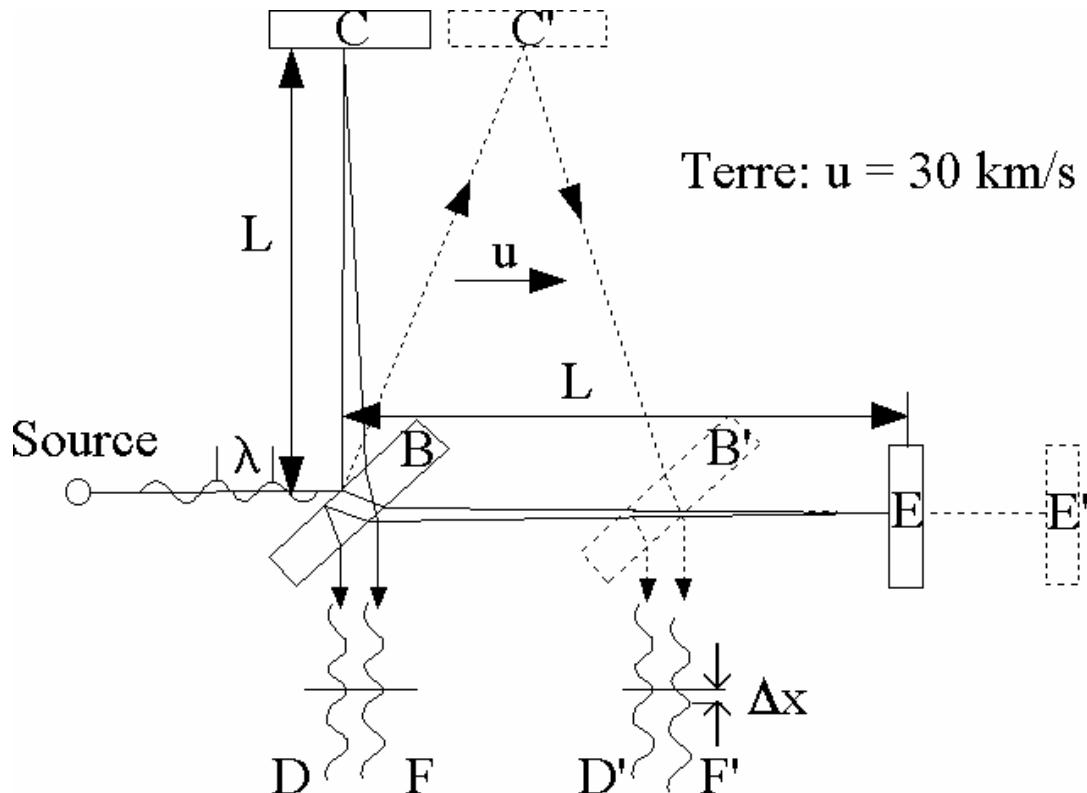


Movable mirror with precise micrometer drive.





## L'expérience de Michelson – Morley (1887) :



*Expérience de Michelson et Morley : les chemins empruntés par les rayons lumineux lors du déplacement de la Terre sont représentés en pointillés.*

*La vitesse de la Terre ajoutée à celle de la lumière aurait dû engendrer des interférences qui ne furent jamais observées.*

Light is a wave and a wave must have a medium in which to travel.

All the other waves we know about required a medium.

Since no medium was apparent between the earth and the sun, it was presumed that this medium was transparent and therefore not readily observable (called “ether”).

This ether was stationnary and filled all of space.

This involved the presumption that there was an absolute reference frame in the universe, and that all the movement of planets and stars was through this ether.

**These presumptions were part of the historical setting of the Michelson Morley experiment.**

With the interferometer which he invented, Michelson found no evidence of the ether.

“The interpretation of these results is that there is no displacement of the interference bands. The result of the hypothesis of a stationnary ether is thus shown to be incorrect (Michelson, 1881).”

The two mirrors are at the same distance D from the beam splitter.

- Way and back to the fixed mirror : (in the same direction as Earth)

$$T_1 = \frac{D}{c+u} + \frac{D}{c-u} = \frac{2Dc}{c^2 - u^2} \approx \frac{2D}{c} \left( 1 + \frac{u^2}{c^2} \right)$$

- Way and back to the movable mirror (perpendicular to the velocity of Earth) :

$$T_2 = \frac{2D}{\sqrt{c^2 - u^2}} \approx \frac{2D}{c} \left( 1 + \frac{u^2}{2c^2} \right)$$

$$T_1 - T_2 = \frac{Du^2}{c^3}$$

Path length difference and interference order :

$$\delta = c(T_1 - T_2) = \frac{u^2}{c^2} D$$

$$; \quad p = \frac{\delta}{\lambda_0} = \frac{u^2}{c^2} \frac{D}{\lambda_0}$$

$$D = 10 \text{ m} ; \quad u = 30 \text{ km.s}^{-1} ; \quad c = 3.10^8 \text{ ms}^{-1} ; \quad \lambda = 500 \text{ nm} \Rightarrow p = 0,2 \text{ fringe}$$

Over a period of about 50 years, the Michelson – Morley experiment was repeated with growing levels of sophistication.

The overall result is a high level of confidence that the wavelength shift is consistent with zero.

	L (cm)	Calculation	Observation	Ratio
Michelson, 1881	120	.04	.02	2
Michelson & Morley 1887	1100	.40	.01	40
Morley & Miller, 1902-04	3220	1.13	.015	80
Illingworth, 1927	200	.07	.0004	175
Joos, 1930	2100	.75	.002	375

## Conceptual framework : relativity

The measurement of absolute velocity is not possible.

Michelson-Morley Experiment

The velocity of light is independent of source or detector velocity - a universal constant.

Sharp double-star images

Aberration of star positions

The Lorentz transformation

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \quad t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The Galilean transformation and the concept of universal time must be abandoned.

Length Contraction

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}} = \frac{L_0}{\gamma}$$

Muon decay in the atmosphere

Time Dilation

$$T = \frac{T_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma T_0$$

Muon decay in the atmosphere

Relativistic Doppler effect.

Fine structure in atomic spectra

Conservation of momentum leads  
to relativistic mass:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Cyclotron frequency

Accelerators

Velocity of light, "c" as speed  
limit of the universe.

Cerenkov radiation

Energy - mass relationship:

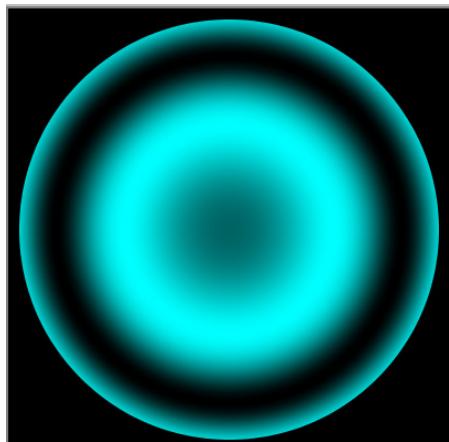
$$E = mc^2$$

Binding energy of nuclei

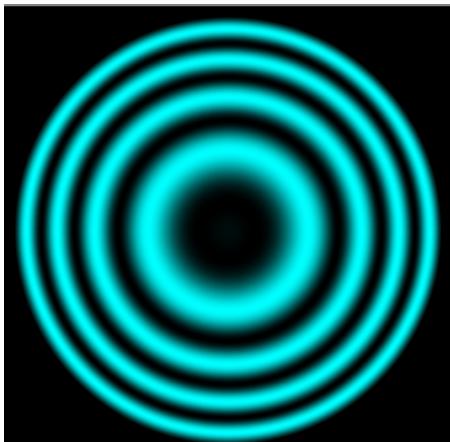
Nuclear fission and fusion.

**Evolution of the circular fringes when the mirror ( $M_2$ ) is moving :**

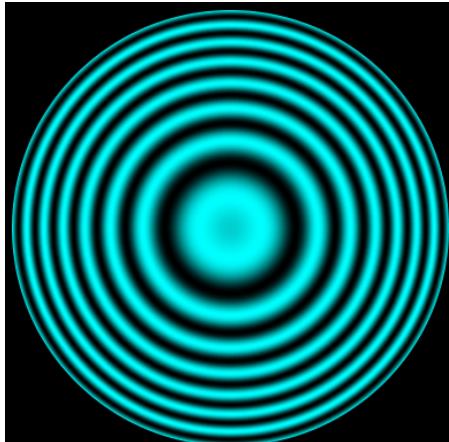
**Animations JJR/Optique ondulatoire/Interférences/Interféromètre de Michelson**



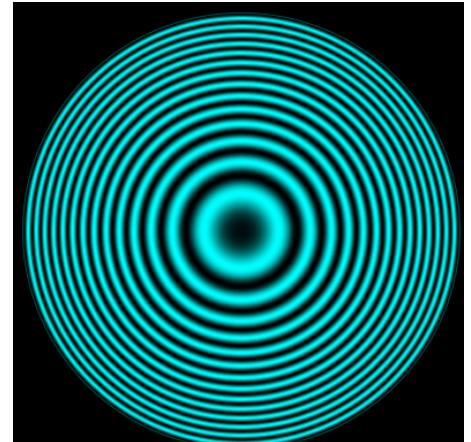
$e = 0,12 \text{ mm}$



$e = 0,4 \text{ mm}$



$e = 0,78 \text{ mm}$



$e = 1,5 \text{ mm}$

$$\delta = 2e \cos(i) = p\lambda_0 = (p_0 - K)\lambda_0$$

## Spectroscopy using the Michelson Interferometer :

Determine the two dominant wavelengths in the sodium light using the Michelson Interferometer

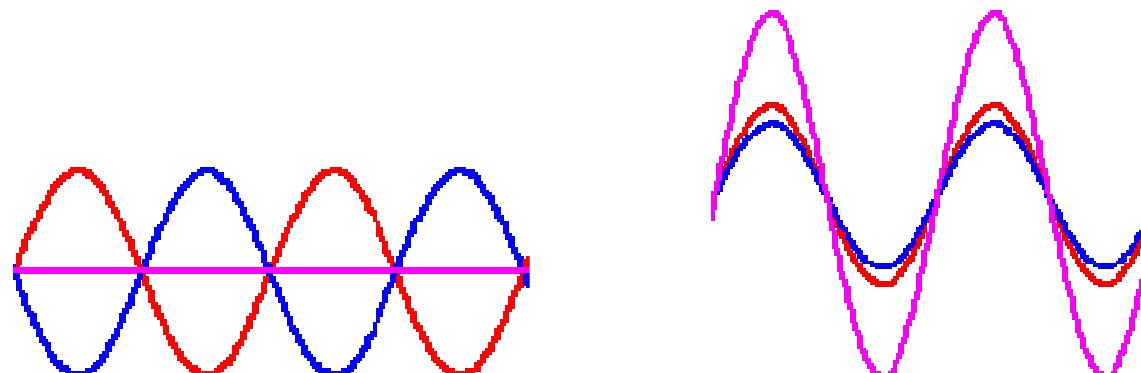
$$\lambda_1 = 589 \text{ nm} \quad \lambda_2 = 589,6 \text{ nm} \quad (\Delta\lambda = 0,6 \text{ nm})$$

At the center of the interference pattern :

$$\delta = 2e \quad ; \quad \Delta\varphi_1 = 2\pi \frac{2e}{\lambda_1} \quad ; \quad \Delta\varphi_2 = 2\pi \frac{2e}{\lambda_2}$$

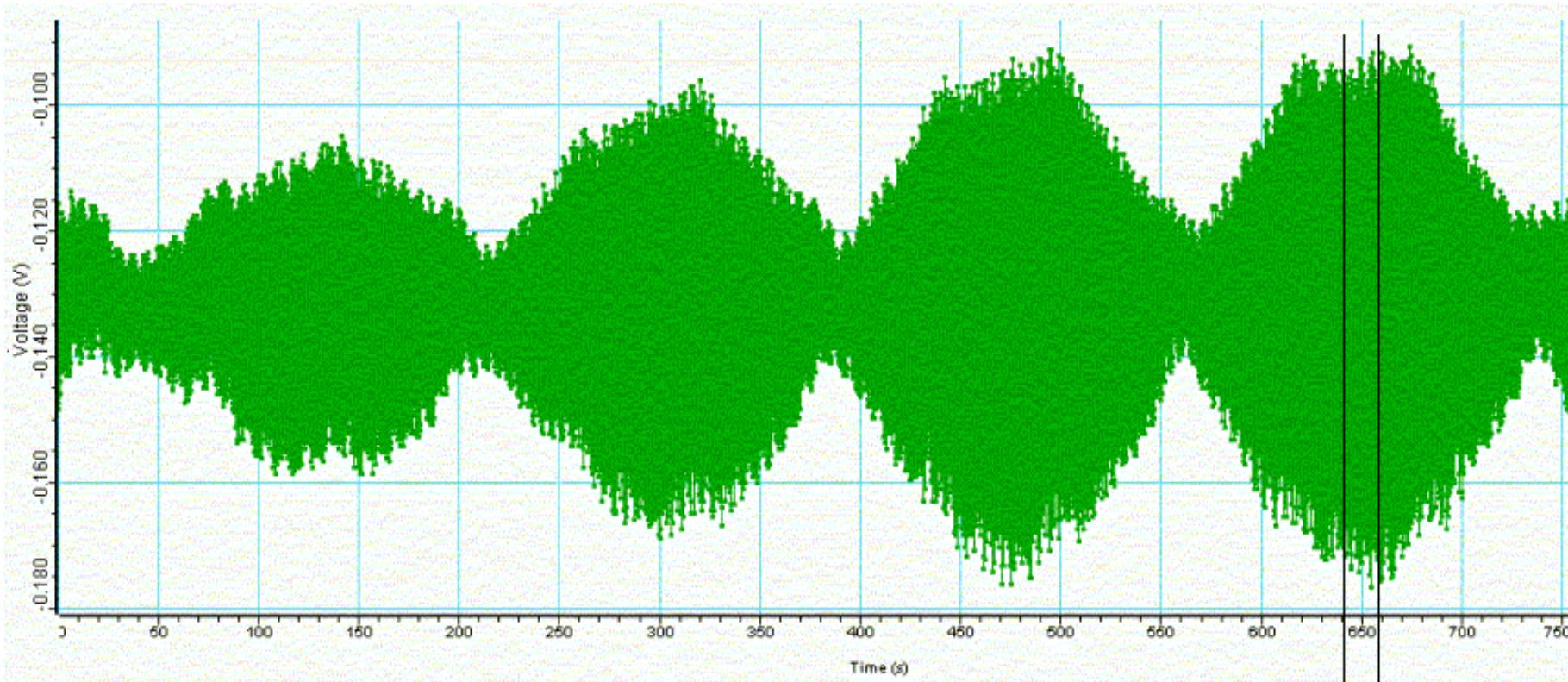
The path lengths traveled by each wave will affect the relative phase of the two waves, so that when they combine, they will combine constructively or destructively.

The images below illustrate constructive and destructive interference with the red and blue waves being the discrete waves and the purple wave representing the combination of the two.



We recorded the intensity of the light going into a detector at the center of the interference pattern as a function of the distance  $e$  between the two mirrors ( $e = vt$ )

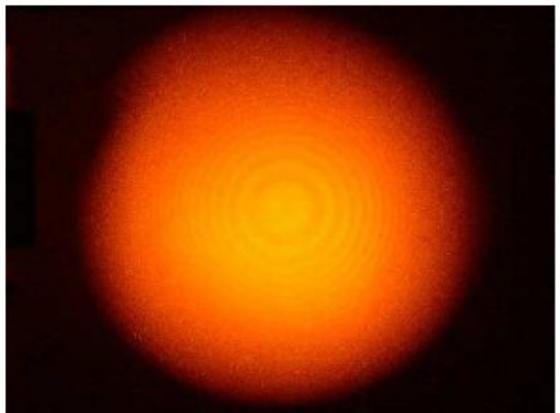
(<http://webphysics.davidson.edu/alumni/BeKinneman/spec/report.htm>)



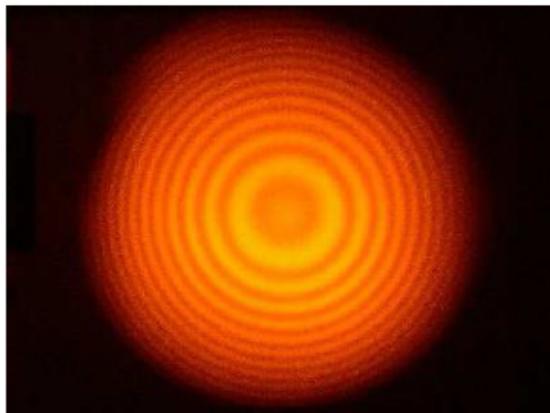
The frequency of the fringes is 5.75 fringes/sec. The beats have a period of 178.17 sec.

The velocity of the movable mirror : 1666.4  $\mu\text{m/sec}$ .

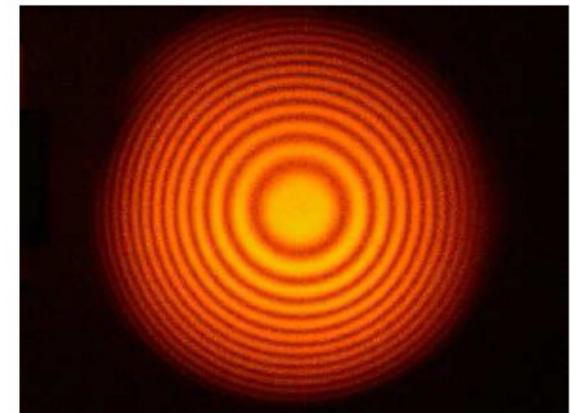
## Vidéo : « Le doublet du sodium : coïncidences et anticoïncidences »



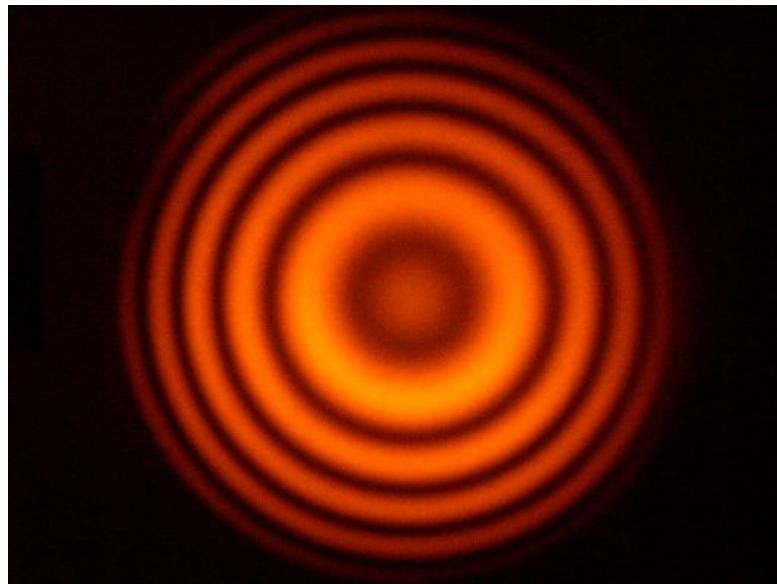
*Weak contrast : destructive interference*



*Medium contrast*



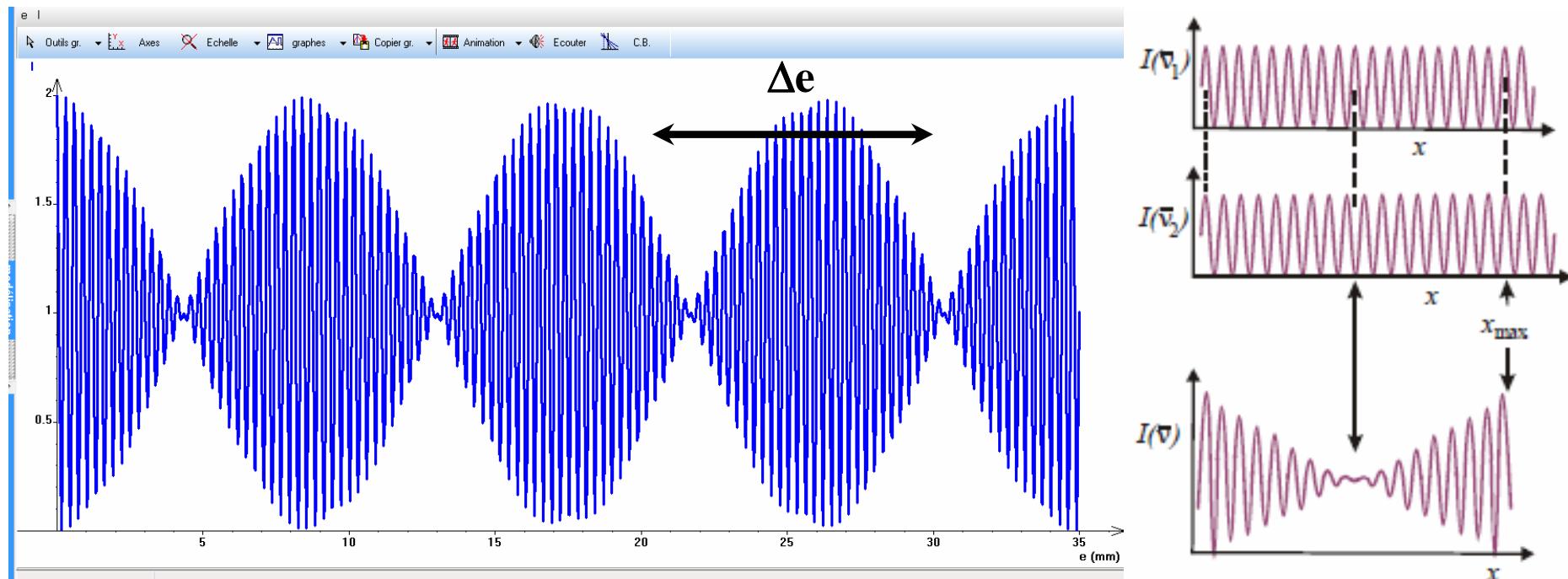
*Neat contrast : constructive interference*



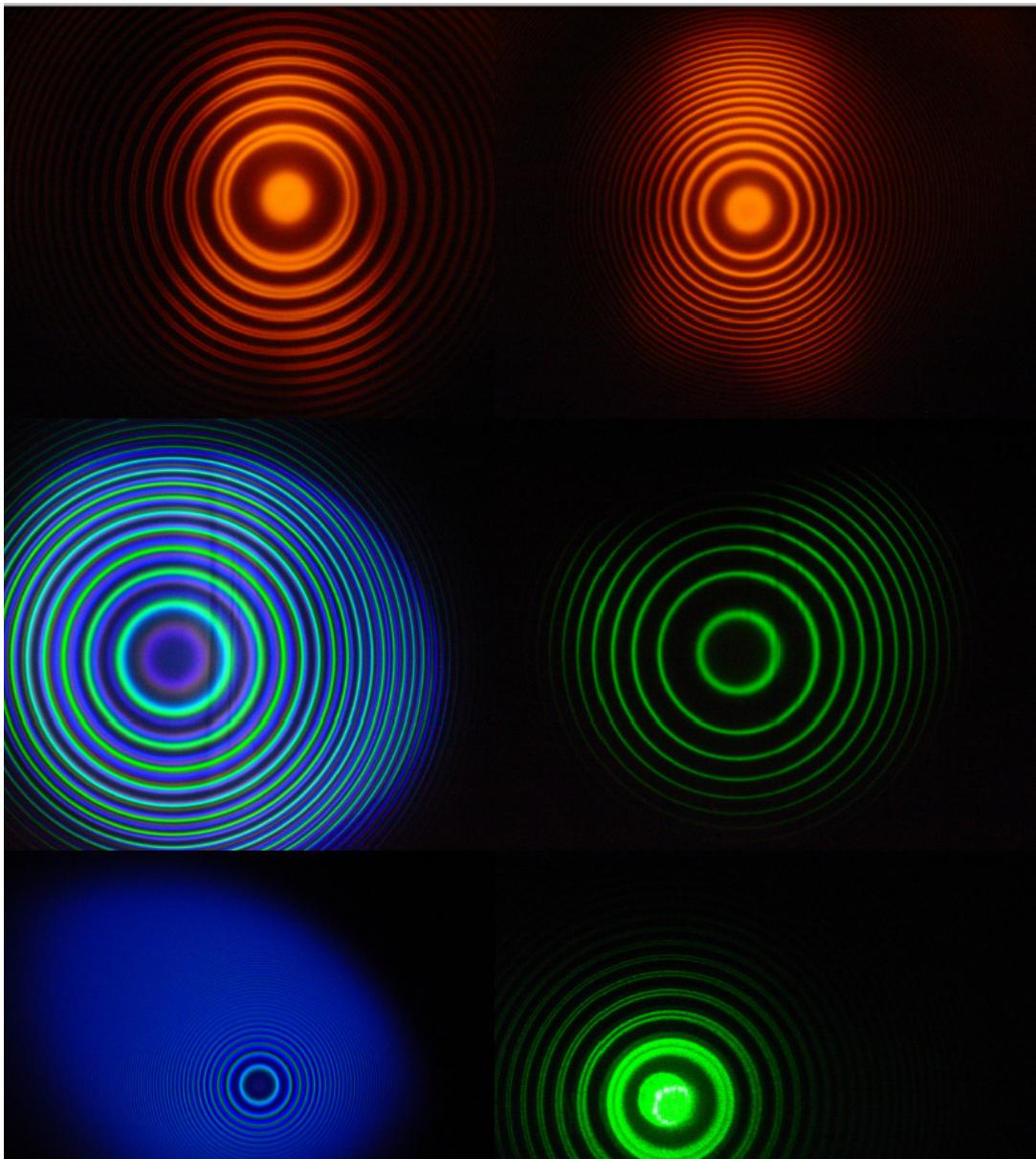
$$I = 2I_0 \left( 1 + \cos\left(\frac{2\pi e \Delta \lambda}{\lambda_0^2}\right) \cos\left(\frac{4\pi e}{\lambda_0}\right) \right) \quad \left( \lambda_0 = \frac{\lambda_1 + \lambda_2}{2} \right)$$

$$\Delta \lambda = \frac{\lambda_0^2}{2\Delta e} \quad (\lambda_0 = 589,3 \text{ nm} ; \Delta \lambda = 0,6 \text{ nm})$$

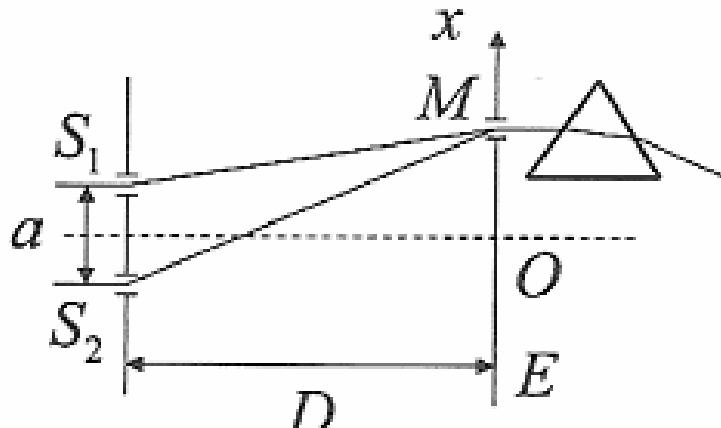
$\Delta e$  : distance between two positions of the mirror ( $M_1$ ) when the light intensity is equal to zero.



File Regressi software (« Battements optiques »)

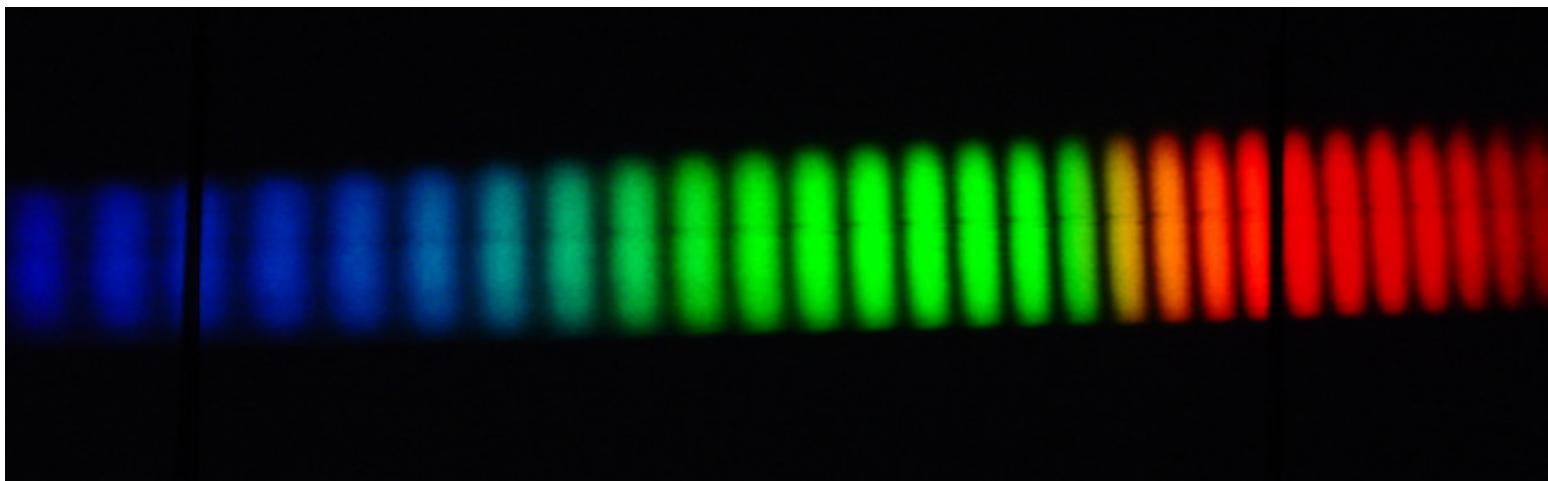


## Young's slits experiment with white light : (Channeled spectrum)



There is a slit in M ( $x = 5$  mm) and a prism is put at the exit.  
Without prism, we see nothing but uniform, apparently white light.  
With prism, the various colours are dispersed on a screen.  
The rainbow produced by the white light is not uniform : it contains dark fringes, showing that the spectrum of the light has been changed (channeled spectrum)

( $a = 1,5$  mm ;  $D = 1,5$  m)





Path length difference :

$$\delta = \frac{ax}{D} = 5 \mu m$$

Condition to have dark fringes : (destructive interferences)

$$\delta = \frac{ax}{D} = 5 \mu m = \left( m + \frac{1}{2} \right) \lambda_0 \quad (\lambda_0 \in [0,4 \mu m, 0,8 \mu m])$$

For :

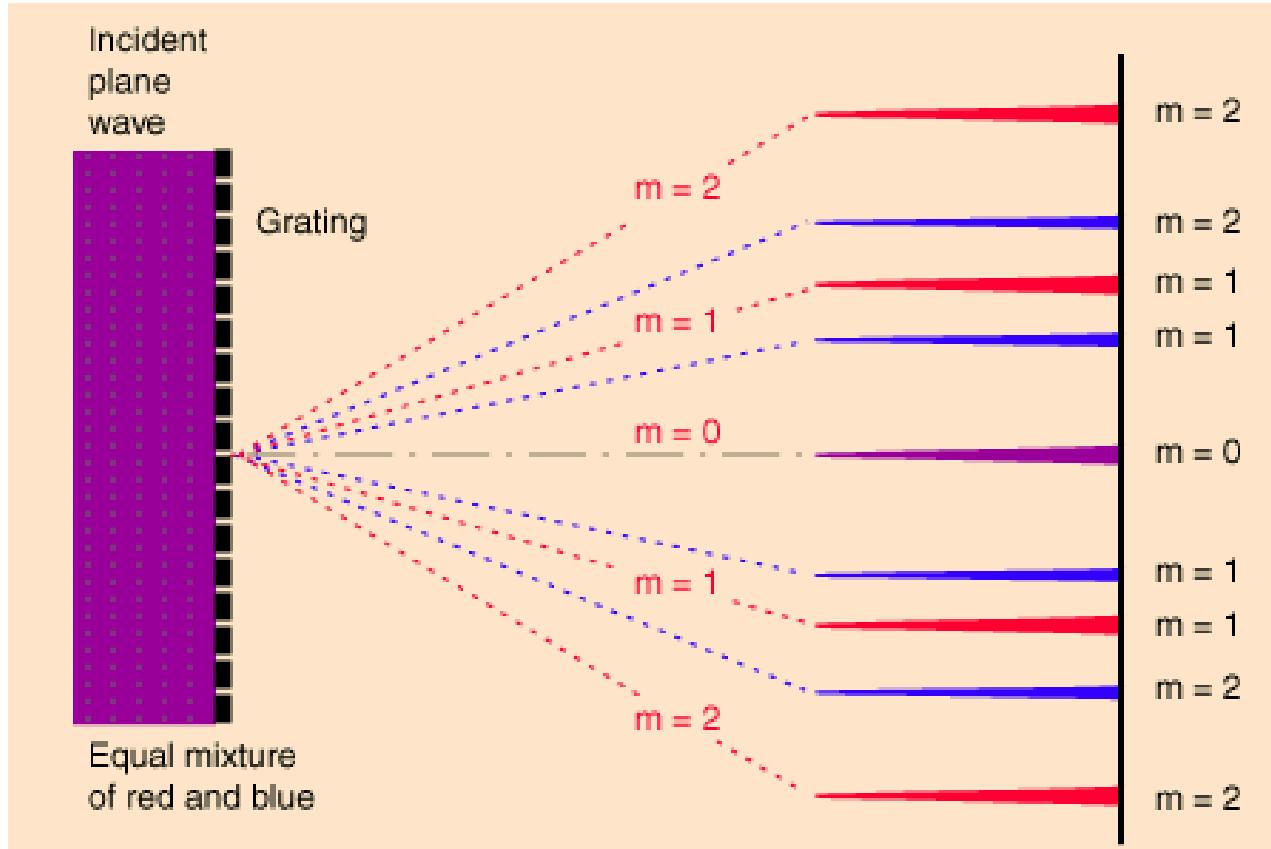
$$\lambda_0 = 0,4 \mu m \Rightarrow m + \frac{1}{2} = 12,5 \quad hence \quad m = 12$$

The different wavelengths given dark fringes are :

Interference order m	12	11	10	9	8	7	6
Wavelength ( $\mu m$ )	0,4	0,43	0,47	0,52	0,59	0,67	0,77

We can see 7 dark fringes.

## Vidéo : « Réseaux de diffraction »



A diffraction grating is an optical component with a periodic structure. It splits and diffracts light into several beams travelling in different directions.

When there is a need to separate light of different wavelengths with high resolution, then a diffraction grating is most often the tool of choice.

A large number of parallel, closely spaced slits constitutes a diffraction grating.

The condition for maximum intensity is the same as that for the double slit or multiple slits.

The intensity maximum is very narrow (providing high resolution for spectroscopic applications).

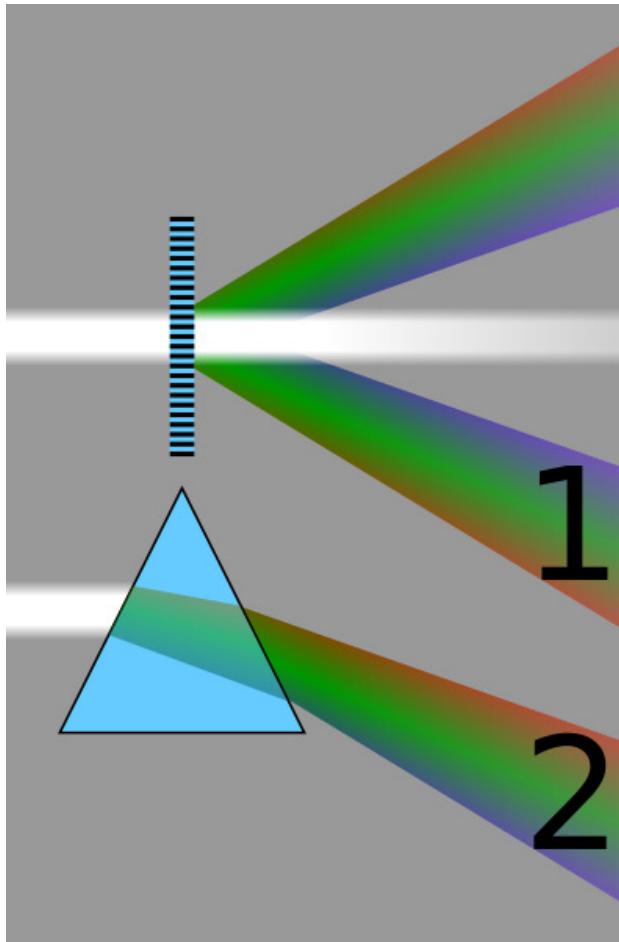
The peak intensities are also much higher for the grating than for the double slit.



A beam of helium-neon laser is diffracted to each side in multiple orders.

Many orders are shown to each side of the direct beam, according to the grating relationship :

$$\sin \theta - \sin \theta_0 = m \frac{\lambda_0}{a}$$



*Comparison of the spectra obtained from a diffraction grating by diffraction (1), and a prism by refraction (2).*

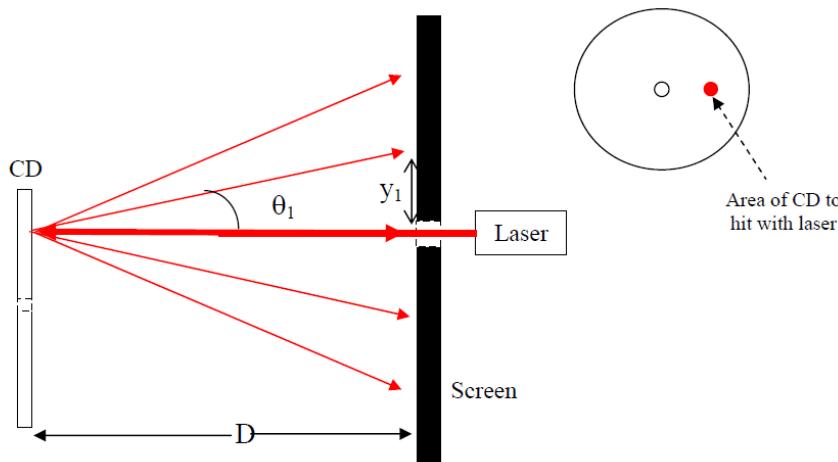
*Longer wavelengths (red) are diffracted more, but refracted less than shorter wavelengths (violet).*



The tracks of a compact disc act as a diffraction grating, producing a separation of the colors of white light.

The nominal track separation on a CD is  $1.6 \mu\text{m}$  (about 625 tracks per millimeter).

For laser beam, this would give a first order diffraction maximum at about  $22^\circ$ .



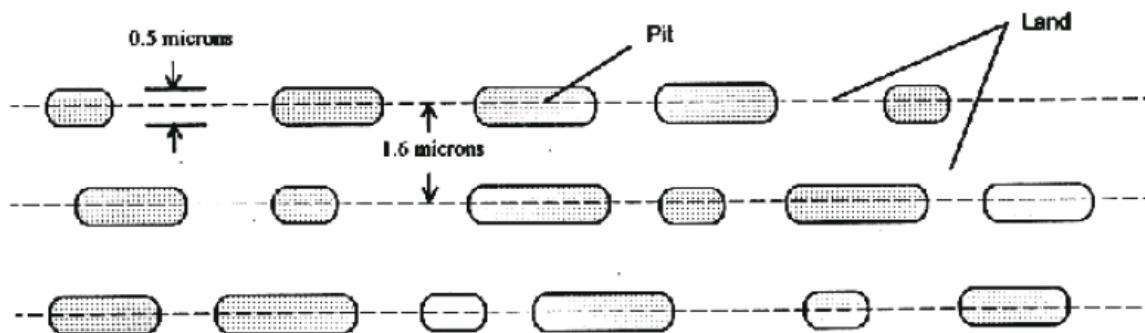
$D$  = distance from CD to screen.

$y_1$  = distance from central beam to 1<sup>st</sup> diffracted order.

$\theta_1$  = angle of 1<sup>st</sup> diffracted order.

$\lambda$  = laser wavelength

$a$  = CD pit spacing =  $1.6 \times 10^{-6}$  meters (shown below)



Geometry of Compact Disc optical data (pit spacing)

# Creating Transmission Gratings from a CD



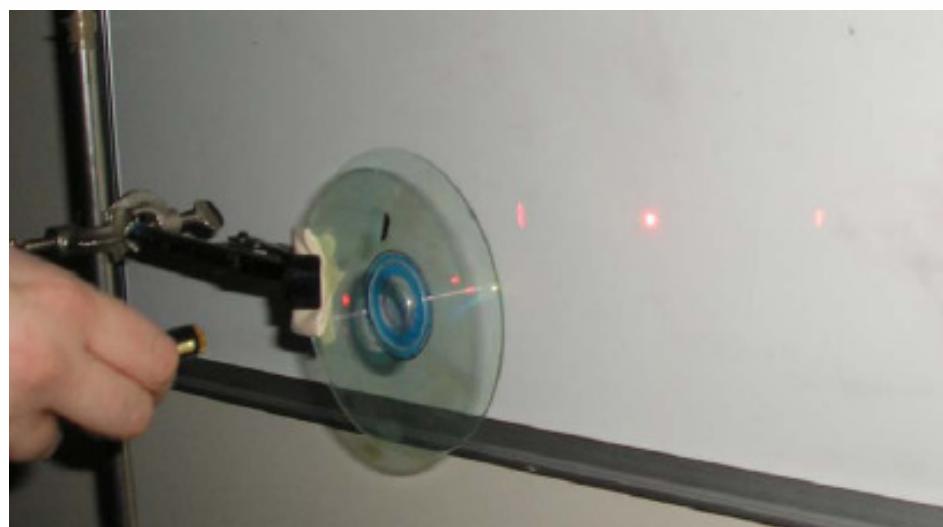
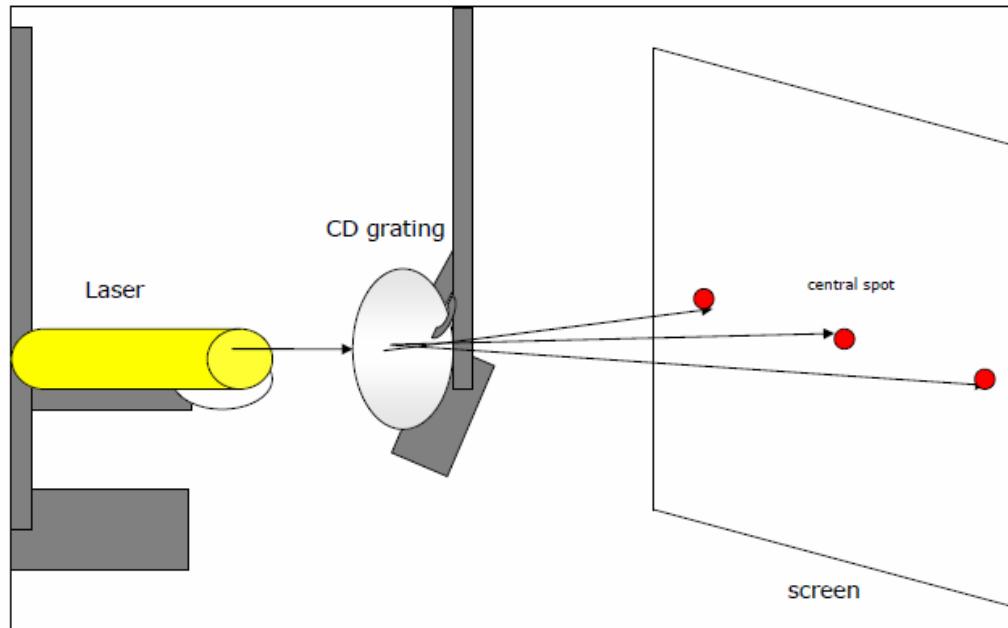
**1: Scratch the label surface of the CD to begin especially with painted labels (Blank CDs work best).**

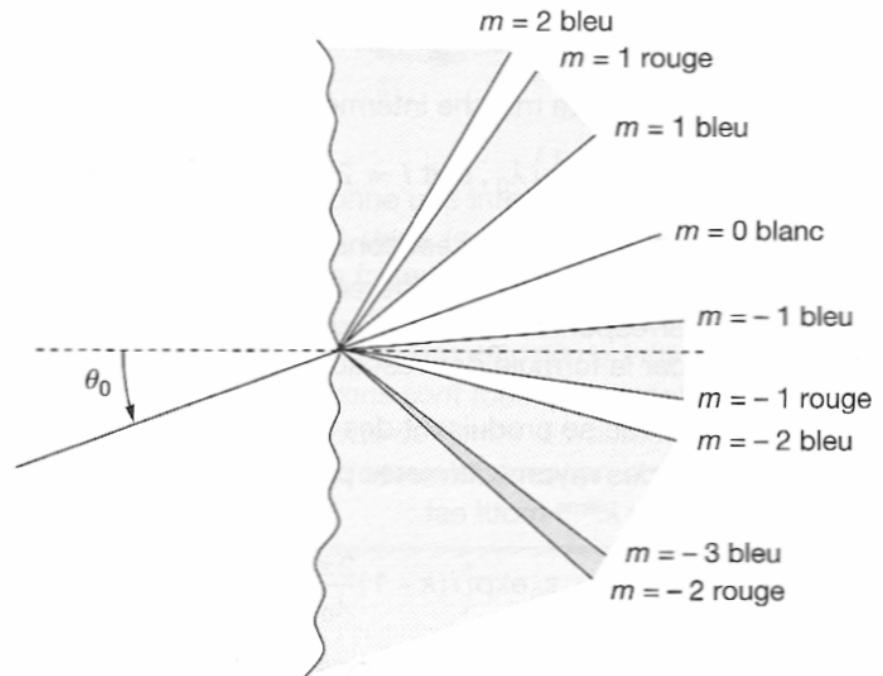
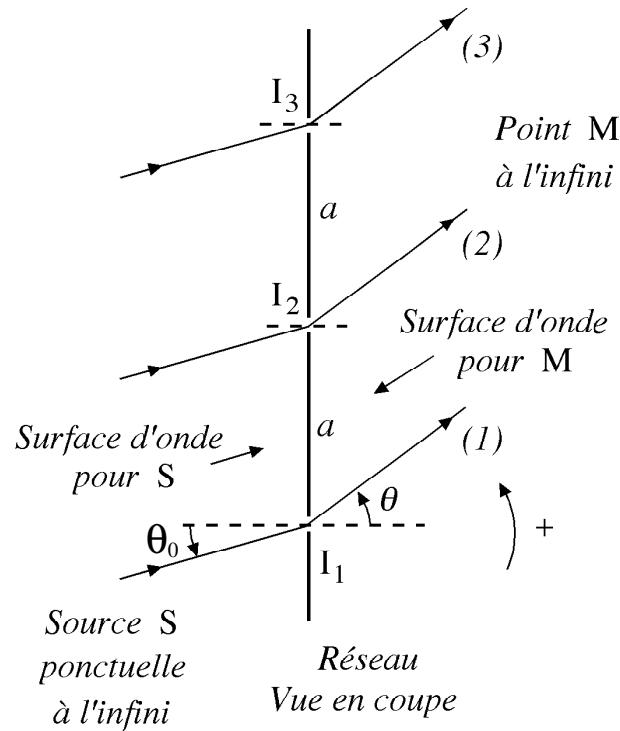


**2: Use Quik-stic or any other tape to peel off the label and reflecting layers.**



**3: CD ready to use as transmission diffraction grating**





$$\sin \theta - \sin \theta_0 = m \frac{\lambda_0}{a}$$

Intensité diffractée : (incidence normale,  $\theta_0 = 0$ )

$$I(\theta) = I_0 \sin c^2 \left( \frac{\pi b \sin \theta}{\lambda} \right) \frac{\sin^2(N\pi a \sin \theta / \lambda)}{N^2 \sin^2(\pi a \sin \theta / \lambda)}$$

With :

$$\underline{s}_{k+1} = \underline{s}_k e^{-i\varphi}$$

The total amplitude is : (geometrical progression with  $e^{-i\varphi}$  as common ratio)

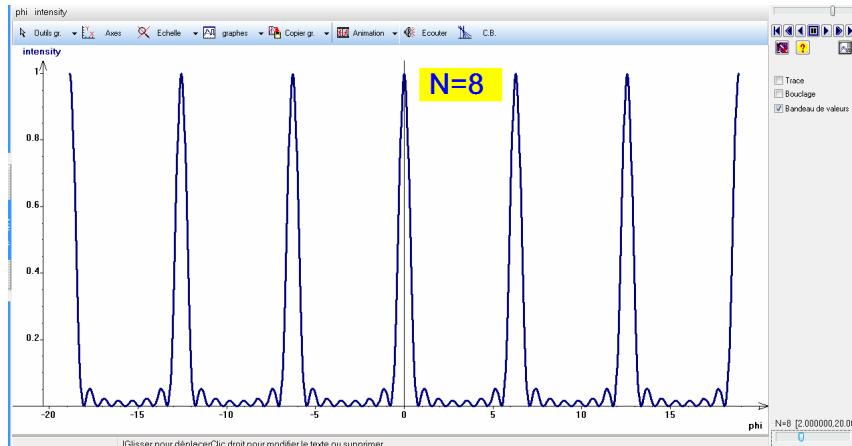
$$\underline{s}_{tot} = \sum_{k=0}^{N-1} \underline{s}_1 e^{-ik\varphi} = \underline{s}_1 \frac{1 - e^{-iN\varphi}}{1 - e^{-i\varphi}}$$

So :

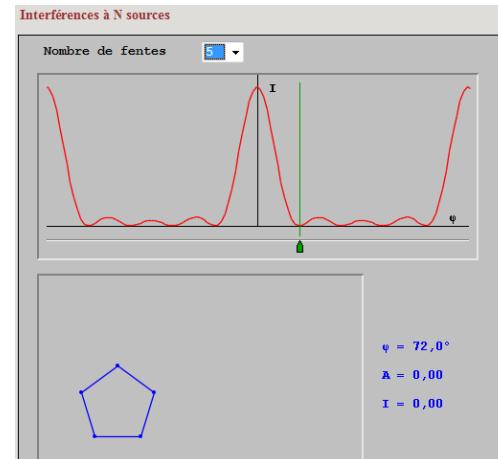
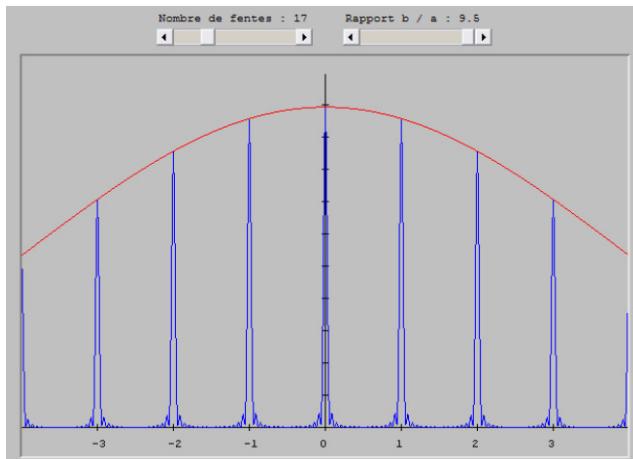
$$\underline{s}_{tot} = \underline{s}_1 \frac{e^{-iN\varphi/2}}{e^{-i\varphi/2}} \frac{e^{iN\varphi/2} - e^{-iN\varphi/2}}{e^{i\varphi/2} - e^{-i\varphi/2}} = \underline{s}_1 \frac{e^{-iN\varphi/2}}{e^{-i\varphi/2}} \frac{\sin(N\varphi/2)}{\sin(\varphi/2)}$$

The intensity is therefore : (normal incidence,  $\theta_0 = 0$ )

$$I(\theta) = I_0 \sin^2 \left( \frac{\pi b \sin \theta}{\lambda} \right) \frac{\sin^2(N\pi a \sin \theta / \lambda)}{N^2 \sin^2(\pi a \sin \theta / \lambda)}$$



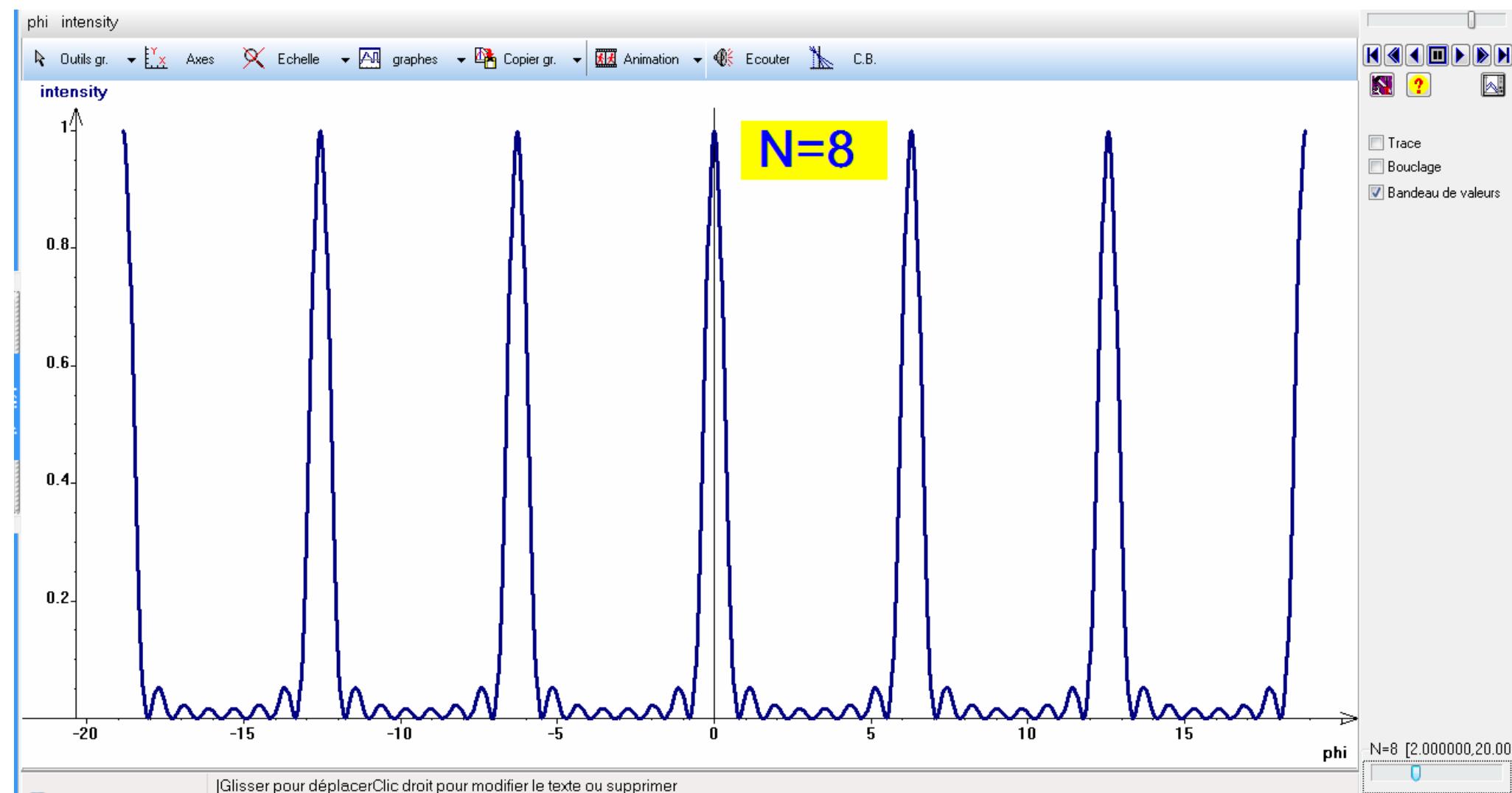
**File Regressi**

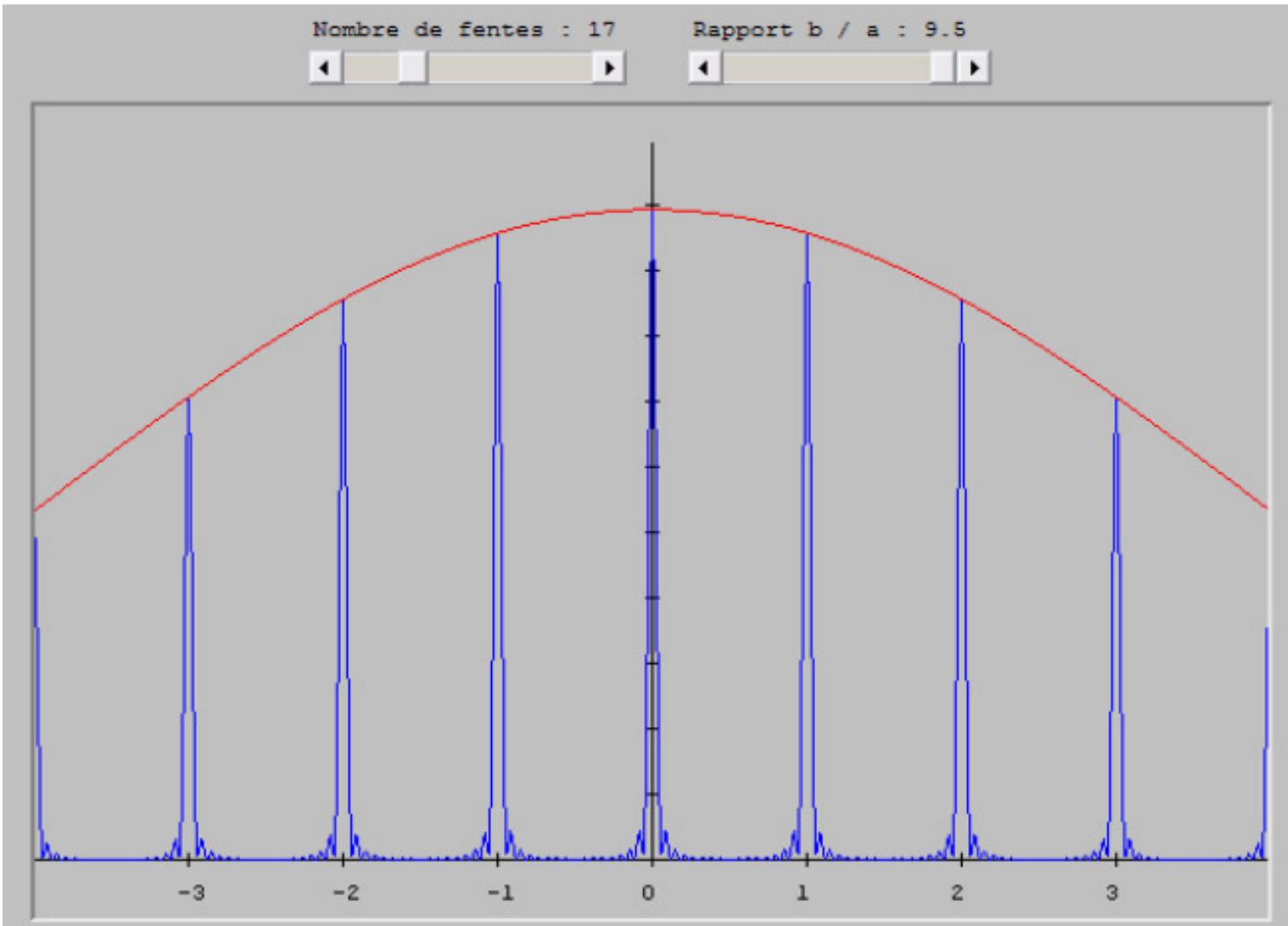


**Animations JJR : Animations JJR/Optique ondulatoire/Diffraction/Réseau plan (principe)**

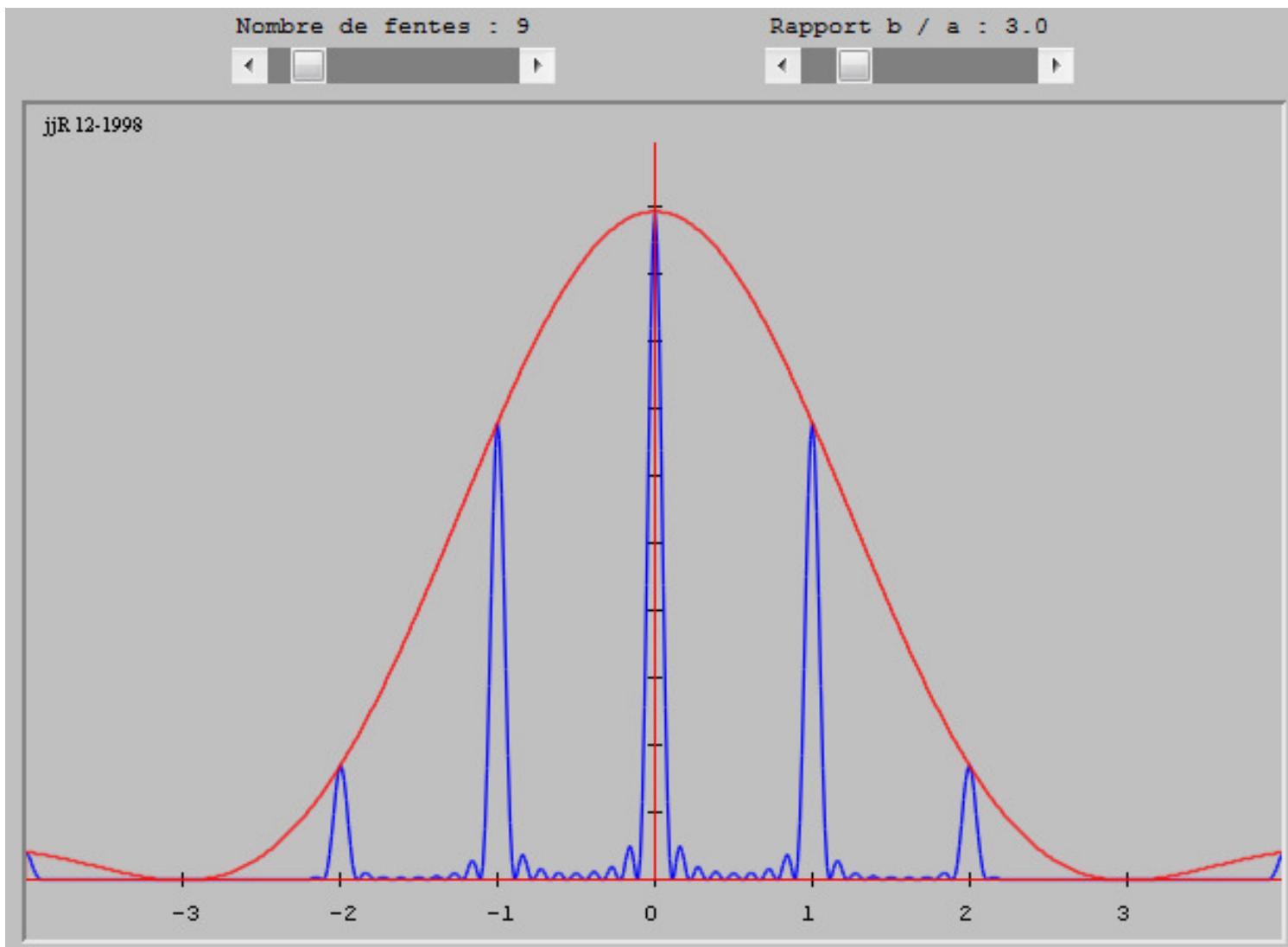
**Animations JJR/Optique ondulatoire/Interférences/Interférences à N sources**

**Animations JJR/Optique ondulatoire/Interférences/Interférences à N sources (Fresnel)**

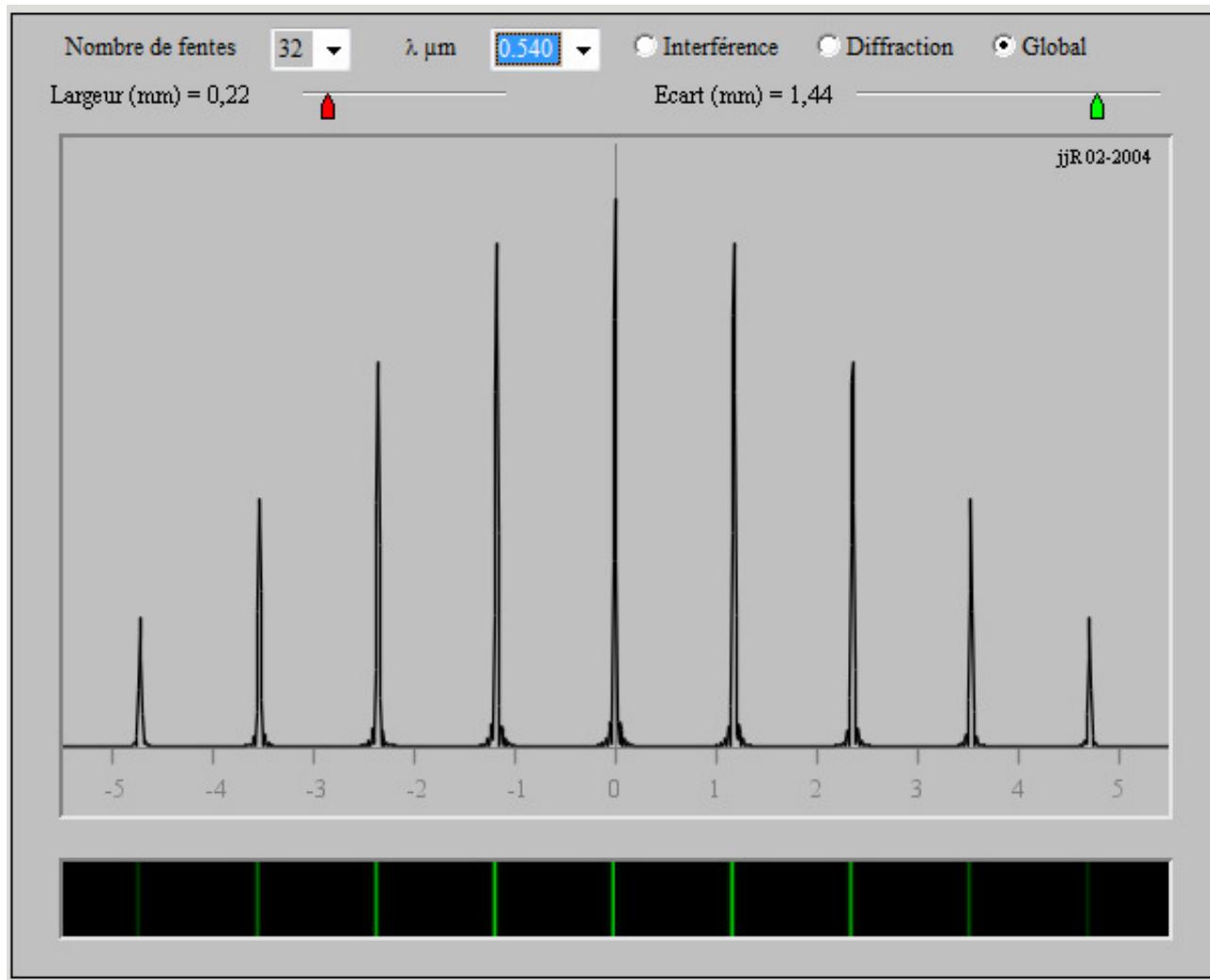




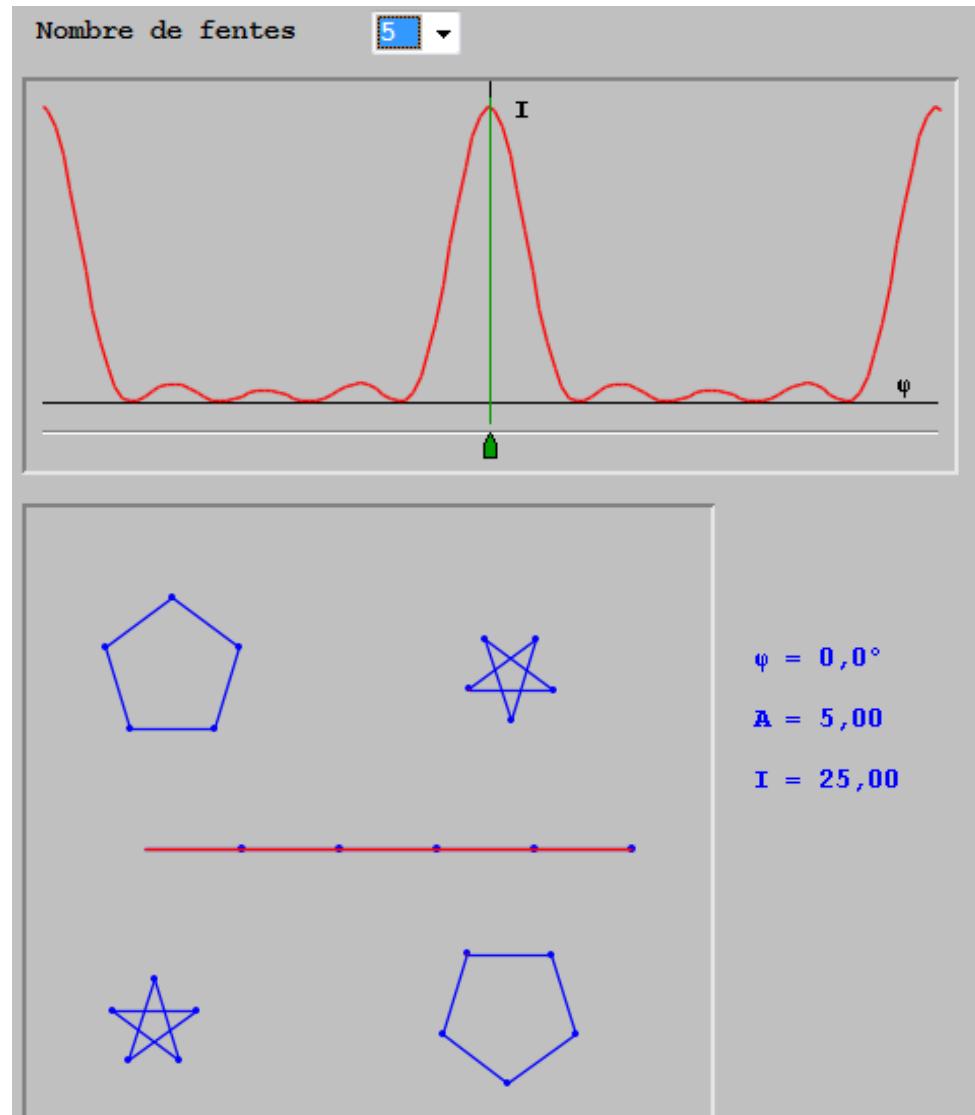
## Animations JJR/Optique ondulatoire/Diffraction/Réseau plan (principe)



## Animations JJR/Optique ondulatoire/Diffraction/Réseau plan (principe)



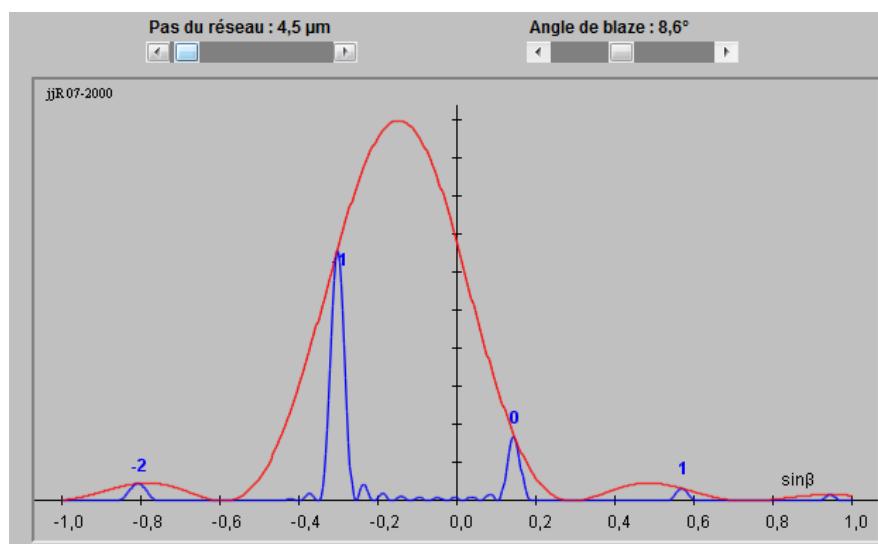
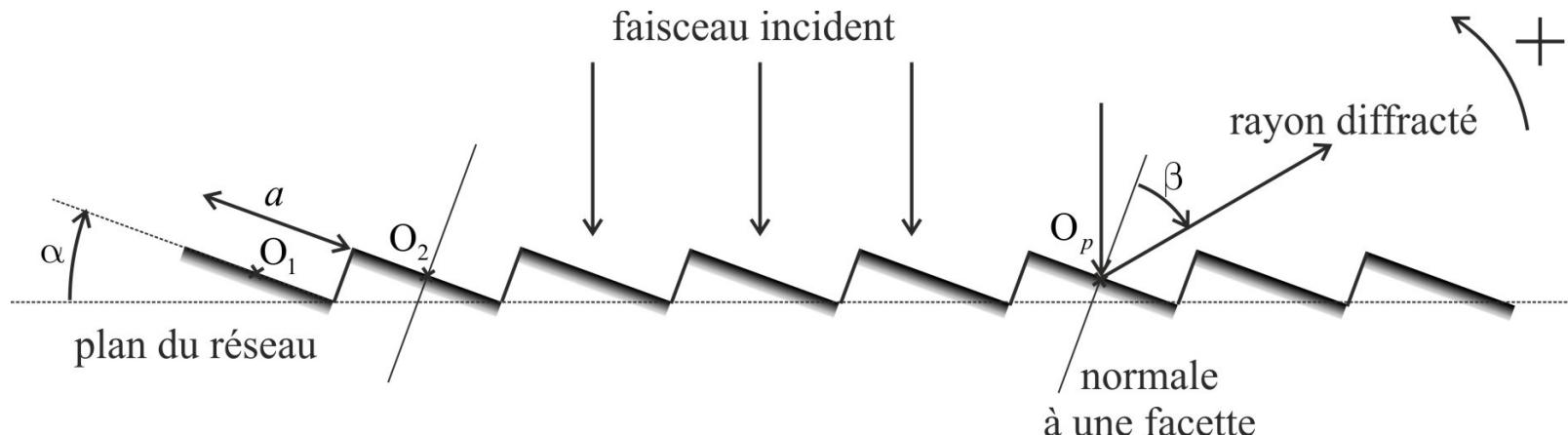
## Animations JJR/Optique ondulatoire/Interférences/Interférences à N sources



## Animations JJR/Optique ondulatoire/Interférences/Interférences à N sources (Fresnel)

## Diffraction grating with a triangular profile (technique is called blazing)

Animation JJR : [Animations JJR/Optique ondulatoire/Diffraction/Réseau blazé](#)



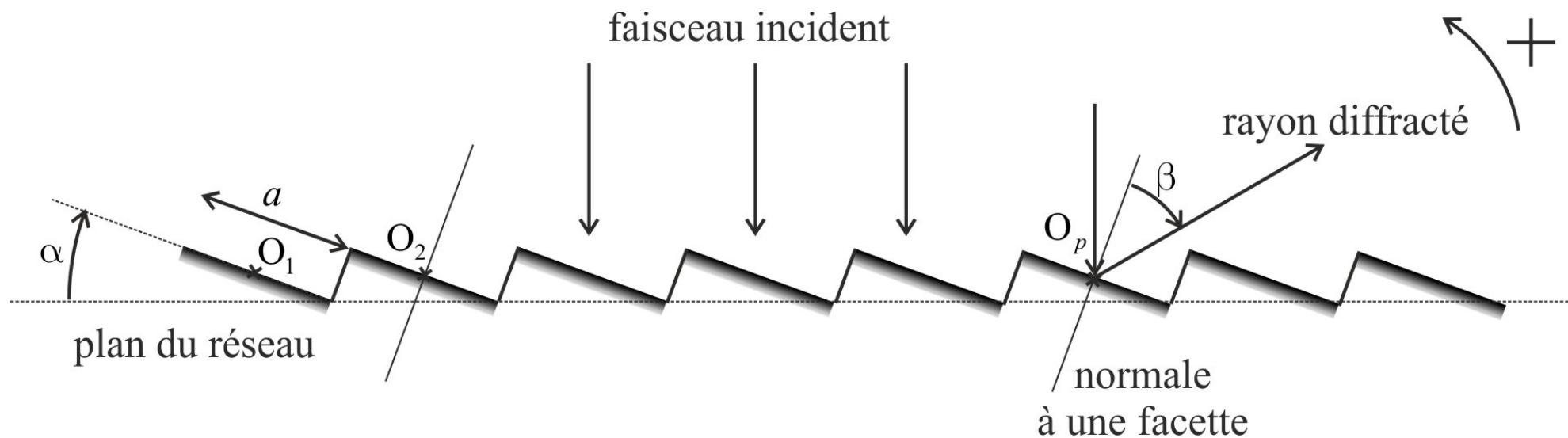
It is possible to concentrate most of the diffracted energy in a particular order for a given wavelength.

This is a dispersif order.

L'animation proposée permet de tracer les deux graphes des fonctions de diffraction (en rouge) et d'interférences (en bleu).

On peut faire varier le pas  $a$  du réseau ainsi que l'angle d'une facette  $\alpha$ .

Quel est l'intérêt de ce réseau comparé à un réseau par transmission constitué de  $N$  fentes équidistantes ?



Le déphasage entre l'onde passée par  $O_1$  et l'onde passée par  $O_p$ , arrivant sous incidence normale et diffractée dans la direction  $\beta$ , est  $p\varphi(\beta)$  où :

$$\varphi(\beta) = \frac{2\pi}{\lambda} \frac{a}{\cos \alpha} \sin(\beta - \alpha)$$

Dans quelle direction observe-t-on des maxima d'intensité ?

Quel est l'effet de la diffraction ?

On peut ajuster les paramètres du réseau de telle sorte que, pour une longueur d'onde donnée (on utilise ici une lumière infrarouge de longueur d'onde égale à  $2 \mu\text{m}$ ), seul le maximum d'ordre  $p$  soit lumineux et tous les autres éteints.

## Un TD expérimental

Pour réaliser l'expérience historique des trous d'Young, on dénude un fil de cuivre très fin et on perce avec application deux trous rapprochés ( $S_1$  et  $S_2$ ) dans une feuille d'aluminium alimentaire. On place ensuite ces trous dans le faisceau lumineux du laser He-Ne du laboratoire, et on observe la figure obtenue suivante, sur un écran à la distance D des trous.

On notera  $R$  les rayons des trous percés (supposés égaux),  $d$  l'écart de leurs centres.

Sur la figure ci-contre on a superposé une règle translucide graduée en millimètre : la lecture des longueurs est donc directe.

L'onde émise par le laser He-Ne est supposée plane et monochromatique ( $\lambda = 632,8 \text{ nm}$ ).

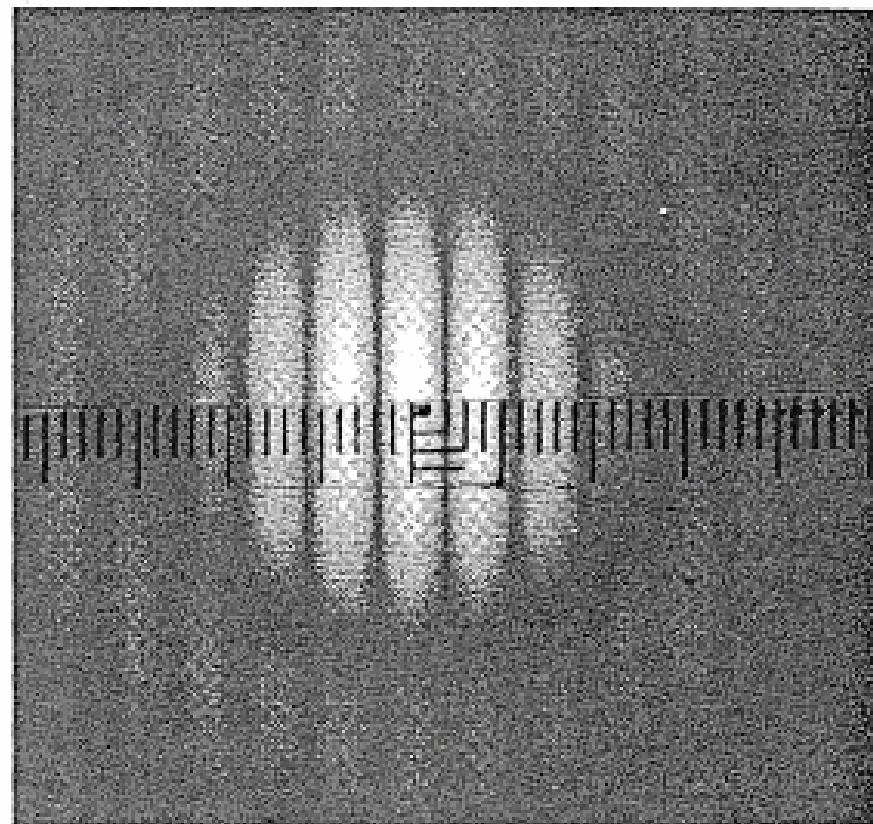
1°) Que signifie le terme "onde plane" ?

2°) Quelle est la couleur du faisceau ?

3°) Sur la figure, on observe deux phénomènes : la diffraction et les interférences.

Identifier clairement la contribution de chaque phénomène.

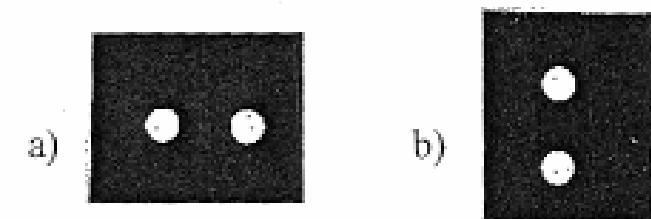
Comment appelle-t-on la figure de diffraction due à la pupille circulaire ?



4°) Pourquoi dit-on que les deux trous constituent des sources secondaires cohérentes ?

5°) Etablir l'expression de la différence de marche  $\delta = S_2M - S_1M$  sur l'écran pour un point  $M(x)$  de l'écran. En appelant  $I_0$  l'intensité lumineuse émise par un trou, donner alors sans démonstration l'expression de l'intensité lumineuse  $I(x)$  en un point  $M$  de l'écran. Justifier alors la forme des franges.

6°) Pour obtenir la figure ci dessus, doit-on placer les trous en position a) ou b) ?



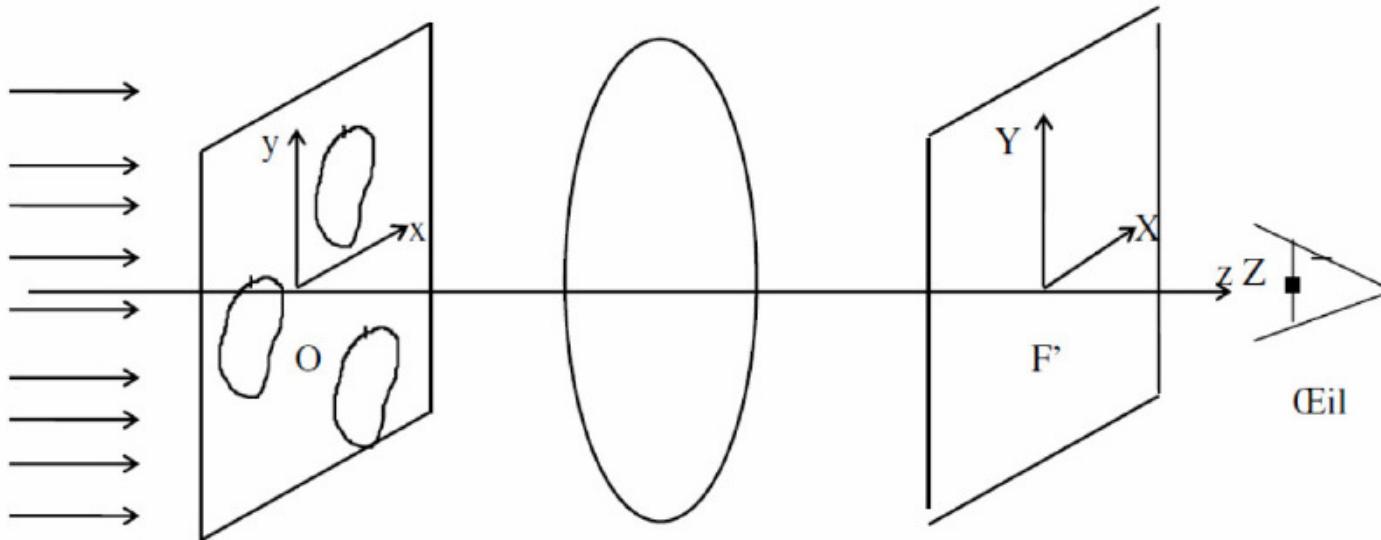
7°) Estimer la valeur de l'interfrange. Comment obtenir une valeur approchée de  $d$  avec  $D = 3,7 \text{ m}$  ?

8°) Qu'observe-t-on si les deux trous ne sont pas de même diamètre ?

9°) a) On effectue une vérification de cette mesure en plaçant la feuille d'aluminium sur la platine d'un microscope. Il est composé de deux lentilles minces convergentes ; l'objectif, de très courte focale (qui fournit de l'objet une image dans le plan focal objet de l'oculaire). Le foyer image de l'objectif est distant de  $\Delta$  du foyer objet de l'oculaire.

Faire un schéma à l'échelle en prenant  $f'_1 = 10 \text{ cm}$ ,  $\Delta = 40 \text{ cm}$  et  $f'_2 = 50 \text{ cm}$  et tracer la marche d'un faisceau lumineux dans le cas où l'image intermédiaire se trouve au foyer objet de l'oculaire.

b) Quel est l'intérêt de ce réglage ?



La pupille diffractante est constituée de deux trous identiques de diamètre D (exprimé en mm dans le logiciel Diffint), décalés de  $\ell$  (en mm) dans une direction inclinée d'un angle noté  $\alpha_l$  (en degré, de  $-90^\circ$  à  $+90^\circ$ , + dans le sens trigonométrique) par rapport à l'horizontale.

Dans le logiciel, on = 0 permet de ne voir que la diffraction due à un seul trou.

Choisir un jeu de valeurs de ( $D$ ,  $\ell$ ,  $\alpha_l$ ) et interpréter complètement la figure observée avec le logiciel.

On effectuera des relevés de distances avec le logiciel.

### Fichier Diffint : « 2 trous »

